NO.

DATE 2017 / 08 / 14.

電磁學公式定律整理(20070803)

Electromagnetc Law, Theorem, formula

§ Eletrostatics

一、真空中靜电學的假說

微分形式　　　　　　積分形式

$$\nabla\cdot\vec{E}=\frac{\rho}{\epsilon_0} \quad \text{——(1)} \qquad \oint_S \vec{E}\cdot d\vec{s}=\frac{Q_T}{\epsilon_0} \quad \text{——(2)}$$

$$\nabla\times\vec{E}=0 \quad \text{——(3)} \qquad \oint_C \vec{E}\cdot d\vec{l}=0 \quad \text{——(4)}$$

物理意義:

(1) 式可知，靜电場不是無散度場 ($\rho\neq0$)

(2) 式是高斯定律 (Gauss's Law) 的另一种形式。意義為在真空中，任何封閉面上电場強度的總通量，等於這封閉面所包含的總电荷量除以 ϵ_0。

(3) 式説明靜电場為無旋場

(4) 式：靜电場強度沿任何封閉路徑的純量線積分等於零。⇒ kirchhoff's voltage law 之數學表示式

($\vec{E}\cdot d\vec{l}$ = 該路徑 dl 之电压)

二、點电荷所造成的电場強度

$$\vec{E} = E_r \hat{r} = \frac{q}{4\pi\epsilon_0 r^2}$$

上式表示帶正电的点电荷的电場強度之方向乃沿著徑向方向指向外，其大小正比於此电荷量而場點與距離的平方成反比。

三、純量电位

$$\vec{E} = -\nabla \vec{V}$$

四、电位差（靜电电压）

$$V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$$

五、电偶極在空間中之电位及电場

電位 $V = \dfrac{\vec{P} \cdot \hat{a}_R}{4\pi\epsilon_0 R^2}$ or $V = \dfrac{q\, d\cos\theta}{4\pi\epsilon_0 R^2}$

電場 $E = -\nabla V = \dfrac{P}{4\pi\epsilon_0 R^3}(\hat{a}_R\, 2\cos\theta + \hat{a}_\theta \sin\theta)$

NO.

DATE / /

六、靜电場中之導體 — 特性及自然邊界條件

(1) 導體內部 $\begin{cases} \rho = 0 \\ \vec{E} = 0 \end{cases}$

(2) 導體表面的邊界條件 (導体 — 真空界面)

$$\begin{cases} E_t = 0 & —① \\ E_n = \dfrac{\rho_s}{\epsilon_0} & —② \end{cases}$$

式① 表式：在導体表面，电場 $\vec{E}$ 的切向分量 E_t 等於零

式② 表式：电場 $\vec{E}$ 在導体 — 真空界面上的垂直分量 E_n 等於導体表面的面电荷密度 ρ_s (or σ) 除以真空的介电係数 ϵ_0 (permitivity)

綜合 (1)、(2) 知，在靜態條件下，導体表面的电場 $\vec{E}$ 恆垂直於表面。也就是說，在靜態條件下，導體的表面是一個等位面，又在導體內部 $\vec{E}=0$，所以整個導体為等电位體。

NO.

DATE / /

七、極化的介电質中之等效电荷分布

1. 定義極化向量 $\vec{P}$ (polarization vector)

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{n\Delta V} \vec{P_k}}{\Delta V} \quad (C/m^2)$$

表示电偶極矩的體密度

2. 極化电荷密度 (polarization charge density) or 束縛电荷密度

$$\begin{cases} \rho_{ps} = \vec{P} \cdot \hat{a}_n & \text{—①} \\ \rho_p = -\nabla \cdot \vec{P} & \text{—②} \end{cases}$$

在場的計算上極化介电質可以用等效的極化表面电荷密度 ρ_{ps} 及等效的極化體电荷密度 ρ_p 來取代。

八、電通密度及介电常数

1. 定義一基本場量 $\vec{D}$：电通密度 (electric flux density) 或电位移 (electric displacement)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

(1) 修正因極化介电所產生之电場影響 $\nabla \cdot \vec{E} = \frac{1}{\epsilon_0}(\rho + \rho_p)$

$$\nabla \cdot \vec{D} = \rho \quad (C/m^2)$$

ρ 為自由的體电荷密度.

2. 電學的高斯定律的另一种形式

$$\oint_S \vec{D}\cdot d\vec{s} = Q_{in} \quad (C)$$

上式表：通過任何封閉面之电位移的總向外通量(或向外的總电通量)，等於包含在這面内的總自由電荷。

3. 當介質的介电特性是線性(linear)且各向同性(isotropic)時，極化向量正比於电場強度，比例常数和电場方向無關.

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

χ_e是無因次量，稱電極化率(electric susceptibility)。
若χ_e和$\vec{E}$無関，則這介电質是線性的，
若和空間座標無関，則介电質為均匀的(homogenous)

4. $\vec{D} = \epsilon_0(1+\chi_e)\vec{E}$
$\quad = \epsilon_0 \epsilon_r \vec{E} = \epsilon \vec{E}$

ϵ_r：稱為介質的相對介电係数(relative permitivity)
或介电常数(dielectric constant)，為一無因次量。

ϵ：稱為絕對介电係数(absolute permitivity)，單位為(F/m)

NO.

DATE / /

九、靜电場的邊界條件（一般介質的界面）

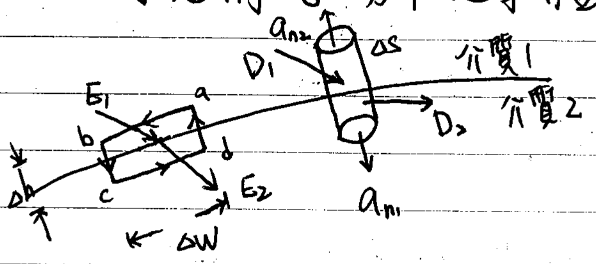
切向分量：$E_{1t}=E_{2t}$ $(V/m)_{SI}$ —— (1)

法向分量：$\hat{a}_{n2}\cdot(\vec{D}_1-\vec{D}_2)=\rho_s$ $(C/m^2)_{SI}$ —— (2)

(1)式表电場$\vec{E}$的切向分量在界面上是連續的

(2)式表當界面存有表面电荷時，$\vec{D}$的法向分量不連續。不連續量等於表面电荷密度。

特例見靜电場中之導體 P.2



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DATE / /

§ Time-Varying Fields and Maxwell's Equation

一、靜电模型及靜磁模型之基本関係

基本関係式	靜电模型	靜磁模型
統轄方程式	$\nabla\times\vec{E}=0$	$\nabla\cdot\vec{B}=0$
	$\nabla\cdot\vec{D}=\rho$	$\nabla\times\vec{H}=\vec{J}$
本構関係式 (線性, 各向同性)	$\vec{D}=\epsilon\vec{E}$	$\vec{H}=\frac{1}{\mu}\vec{B}$

二、法拉第电磁感应定律

1. 电磁感应的基本假說

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

在磁通密度隨時間而变的区域中(自由空間或介質中)，电場強度不保守，而且亦不能將之寫成純量电位的梯度.

2. 時变磁場中的靜止迴路

$$V=-\frac{d\Phi_B}{dt}=-\frac{d}{dt}\oint_s\vec{B}\cdot d\vec{s}$$ —法拉第电磁感应定律—

上式說明在靜止迴路中感应而生之电动勢，等於穿过迴路所圍面積的磁通量之時变率之負值.
其中負號表示在迴路中，由於感应电动勢而產生的电流將沿著抗拒磁通变化的發生方向流动此即冷次定律.

NO.

DATE / /

(1) 應用：变压器

① $M_{12}=k\sqrt{L_1L_2}\le\sqrt{L_1L_2}$

兩線圈互感係数小於等於兩線圈自感係数乘積之平方根，其中k為耦合係数(coefficient of coupling)

三、在靜磁場中的運動導體

1. 磁通切割电動勢(flux-cutting emf)或動生电動勢(motional emf).

$$V'=\int_{all}(\vec{v}\times\vec{B})\cdot d\vec{l}\ \left(=\int_{all}\frac{\vec{F}_B}{q}\cdot d\vec{l}\right)$$

四、在時变磁場中的移动迴路

$$\int_{all}\vec{E}'\cdot d\vec{l}=-\int_s\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}+\int_{all}(\vec{v}\times\vec{B})\cdot d\vec{l}\ \left(=-\frac{d\Phi_B}{dt}\right)$$

其中 $\vec{E}'=\frac{\vec{F}}{q}=(\vec{E}+\vec{v}\times\vec{B})$

右端第一項稱(变压器)电動勢，第二項稱動生电动勢

NO.

DATE / /

五、馬克斯威爾方程式(Maxwell's equations)

微分型式	積分型式	
$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	$\oint_c \vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}$	法拉第感应定律
$\nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$	$\oint_c \vec{H}\cdot d\vec{l}=\vec{J}_f+\int_s\frac{\partial\vec{D}}{\partial t}\cdot ds$	安培-馬克斯威爾定律
$\nabla\cdot\vec{D}=\rho$	$\oint_s \vec{D}\cdot d\vec{s}=Q_{in}$	电學 高斯定律
$\nabla\cdot\vec{B}=0$	$\oint_s \vec{B}\cdot d\vec{s}=0$	磁學 高斯定律

電磁波公式定律整理(20070804)

E.M.W. Law, Theorem, formula

§ Plane Electromagnetic Waves

一、波動方程式

1. $\left(\nabla^2-\mu\epsilon\frac{\partial^2}{\partial t}\right)\begin{bmatrix}\vec{E}\\ \vec{H}\end{bmatrix}=\begin{bmatrix}\frac{1}{\epsilon}\nabla\rho_v+\mu\frac{\partial\vec{J}}{\partial t}\\ -\nabla\times\vec{J}\end{bmatrix}$ 時域型 wave eq.

2. $(\nabla^2+k^2)\begin{bmatrix}\vec{E}\\ \vec{H}\end{bmatrix}=\begin{bmatrix}\frac{1}{\epsilon}\nabla\rho_v+\mu j w\vec{J}\\ -\nabla\times\vec{J}\end{bmatrix}$ 頻域型 Helmholtz's eq.

3. $(\nabla^2+k^2)\begin{bmatrix}\vec{A}\\ \vec{V}\end{bmatrix}=\begin{bmatrix}-\mu\vec{J}\\ -\frac{\rho_v}{\epsilon}\end{bmatrix}$ 向量磁位與純量电位之頻域型 Helmholtz's eq.

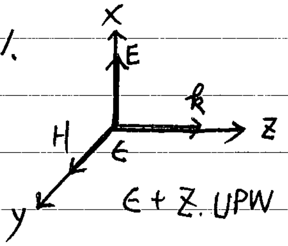
二、波印亭定理

1. 數學式：$\oint_s(\vec{E}\times\vec{H})\cdot(-\vec{ds})=\frac{\partial}{\partial t}\int_v\frac{1}{2}\epsilon E^2dv+\frac{\partial}{\partial t}\int_v\frac{1}{2}\mu H^2dv+\int_v\sigma E^2dv$
2. 物理意義：總能量守恆
3. 瞬時型波印亭向量：$\vec{P}(\vec{R},t)=\vec{E}(\vec{R},t)\times\vec{H}(\vec{R},t)$
4. 平均波印亭向量：$\vec{P}_{av}(\vec{R},t)=\frac{1}{2}\mathrm{Re}[\vec{E}(\vec{R})\times\vec{H}^*(\vec{R})]$

NO.

DATE / /

三、均勻平面波



1. 完全介質 + z方向 均勻平面波

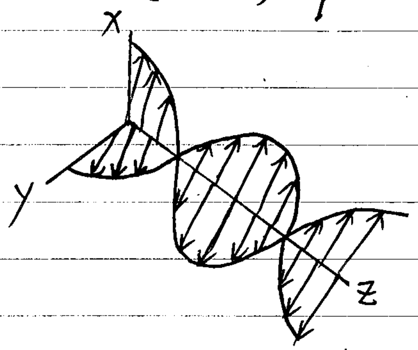
由 $\nabla^2 \vec{E} + k^2 \vec{E} = 0$ 及 $\vec{E} \triangleq \hat{a}_x E_x(z)$

ϵ + Z UPW 求出 $\begin{cases} \vec{E} = \hat{a}_x E_0 e^{-jkz} \\ \vec{H} = \hat{a}_y \dfrac{kE_0}{\omega\mu} e^{-jkz} \end{cases}$, $\eta = \dfrac{\omega\mu}{k} = \sqrt{\dfrac{\mu}{\epsilon}}$

$\eta_0 = 120\pi = 377\ (\Omega)$

真空中本質阻抗

2. 時域: $\begin{cases} \vec{E} = \hat{a}_x E_0 \cos(\omega t - kz) \\ \vec{H} = \hat{a}_y \dfrac{E_0}{\eta} \cos(\omega t - kz) \end{cases}$



$\vec{E}$	頻域: 等相位, 沒有相差 (in phase)
$\vec{H}$	時域: E_{max} 時 H_{max}

3. 導介質中的均勻平面波

通式	完全介質: $\sigma = 0$	良介質 $\sigma \ll \omega\epsilon$	良導体 $\sigma \gg \omega\epsilon$
$\alpha = \omega\sqrt{\mu\epsilon}\sqrt{\dfrac{\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}-1}{2}}$	$\alpha = 0$	$\alpha \cong \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\epsilon}}$	$\alpha = \sqrt{\pi f \mu \sigma}$
$\beta = \omega\sqrt{\mu\epsilon}\sqrt{\dfrac{\sqrt{1+(\frac{\sigma}{\omega\epsilon})^2}+1}{2}}$	$\beta = \omega\sqrt{\mu\epsilon}$	$\beta \cong \omega\sqrt{\mu\epsilon}$	$\beta = \sqrt{\pi f \mu \sigma}$
			$\delta = \dfrac{1}{\sqrt{\pi f \mu \sigma}}$
		$\eta_c \cong \sqrt{\dfrac{\mu}{\epsilon}}$	$\eta_c = (1+j)\sqrt{\dfrac{\pi f \mu}{\sigma}}$
		$v_p \cong \dfrac{1}{\sqrt{\mu\epsilon}}$	$v_p = 2\sqrt{\dfrac{\pi f}{\mu\sigma}}$
			$\lambda = 2\sqrt{\dfrac{\pi}{f\mu\sigma}}$

NO.
DATE / /

四、非均匀平面波 ($\sigma \neq 0$)

1. 定義：$\sigma = 0$ 完全介質

$\sigma \neq 0$ 非完全介質

$\sigma \not\to \infty$ 良導体 good conductor

$\sigma \to \infty$ 完全導体 perfect conductor

2. 物理观念：电磁波在良導体中急劇公式衰減，故只存在於表面

集膚效应 (skin effect)
$\begin{cases} \delta_s: \text{skin depth 集膚深度} & \delta_s = \dfrac{1}{\sqrt{\pi f \mu \sigma}} = \sqrt{\dfrac{2}{\omega \mu \sigma}} \\ R_s: \text{surface resistivity 表面电阻} & R_s = \sqrt{\dfrac{\pi \mu f}{\sigma}} \end{cases}$

3. 平面波基本方程 $\xrightarrow{\text{解}}$ dispersion relation $\tilde{k}^2 = \omega^2 \mu \tilde{\epsilon}$

$$\left[\nabla^2 - \mu\epsilon(j\omega)^2 - \mu\sigma(j\omega)\right]\begin{bmatrix}\vec{E}(\vec{r})\\ \vec{H}(\vec{r})\end{bmatrix} = 0$$

4. 相速與群速之關係式：$V_g = \dfrac{V_p}{1 - \dfrac{\omega}{V_p}\dfrac{dV_p}{d\omega}}$

五、正向入射

1. 介質與導体界面之正向入射：

入射波	透射波	反射波	合成波
$\vec{E}_i(z) = \hat{a}_x E_0 e^{-jkz}$	$E_{PEC} = 0$	$\vec{E}_r = -\hat{a}_x E_0 e^{+jkz}$	$\vec{E}(z) = \hat{a}_x E_0(-2j)\sin kz$
$\vec{H}_i(z) = \hat{a}_y \frac{E_0}{\eta} e^{-jkz}$	$\vec{H}_{PEC} = 0$	$\vec{H}_r = \hat{a}_y \frac{E_0}{\eta} e^{+jkz}$	$\vec{H}(z) = \hat{a}_y \frac{E_0}{\eta} 2\cos kz$

2. 介質與介質界面之正向入射：

① 反射係數：$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

透射係數：$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$

關係式 $\boxed{1 + \Gamma = \tau}$

② 和电場與和磁場公式：

$$\vec{E}_1(z) = \hat{a}_x E_{i0}(\tau e^{-jk_1 z} + \Gamma\, 2j \sin k_1 z)$$

$$\vec{H}_1(z) = \hat{a}_y \frac{E_{i0}}{\eta_1}(\tau e^{-jk_1 z} - \Gamma\, 2\cos k_1 z)$$

③ 电磁場特性：

駐波比： $S \triangleq \dfrac{Max|E|}{Min|E|} = \dfrac{1+|\Gamma|}{1-|\Gamma|}$ 或 $|\Gamma| = \dfrac{S-1}{S+1}$

入射阻抗： $\eta_{in} = \eta_1 \dfrac{\eta_2 + \eta_1 j\tan k_1 l}{\eta_1 + \eta_2 j\tan k_1 l}$

六、斜向入射

1. 介質與導体界面之TE波斜向入射

入射波	反射波	合成波
$\vec{E_i} = \hat{a}_y E_0 e^{-j(k_x x + k_z z)}$	$\vec{E_r} = -\hat{a}_y E_0 e^{-j(k_x x - k_z z)}$	$\vec{E}(x,z) = \hat{a}_y E_0 e^{-jk_x x}(-2j)\sin k_z z$ —①
$\vec{H_i} = (-\hat{a}_x \frac{E_0}{\eta}\cos\theta + \hat{a}_z \frac{E_0}{\eta}\sin\theta) e^{-j(k_x x + k_z z)}$	$\vec{H_r} = (-\hat{a}_x \frac{E_0}{\eta}\cos\theta - \hat{a}_z \frac{E_0}{\eta}\sin\theta) \cdot e^{-j(k_x x - k_z z)}$	$\vec{H}(x,z) = -\hat{a}_x \frac{E_0}{\eta}\cos\theta e^{-jk_x x} 2\cos(k_z z) + \hat{a}_z \frac{E_0}{\eta}\sin\theta e^{-jk_x x}(-2j)\sin(k_z z)$ —②

由①、②式 ⇒ 平均功率 $\vec{P}_{av}(z) = \hat{a}_x \dfrac{2E_0^2}{\eta}\sin^2 k_z z \cdot \sin\theta$.

2. 介質與導体界面之TM波斜向入射

入射波	反射波	合成波
$\vec{E_i} = (\hat{a}_x E_0\cos\theta - \hat{a}_z E_0\sin\theta) e^{-jk_x x} e^{-jk_z z}$	$\vec{E_r} = (-\hat{a}_x E_0\cos\theta - \hat{a}_z E_0\sin\theta) e^{-jk_x x} e^{+jk_z z}$	$\vec{E}(x,z) = \hat{a}_x E_0\cos\theta e^{-jk_x x}(-2j)\sin k_z z + \hat{a}_z E_0(-1)\sin\theta e^{-jk_x x} 2\cos k_z z$ —①
$\vec{H_i}(z) = \hat{a}_y \frac{E_0}{\eta} e^{-jk_x x} e^{-jk_z z}$	$\vec{H_r} = \hat{a}_y \frac{E_0}{\eta} e^{-jk_x x} e^{+jk_z z}$	$\vec{H}(x,z) = \hat{a}_y \frac{E_0}{\eta} e^{-jk_x x} 2\cos k_z z$ —②

由①、②式 ⇒ 平均功率 $\vec{P}_{av}(z) = \hat{a}_x \dfrac{2E_0^2}{\eta}\sin\theta\cos^2 k_z z$

3. 介質與介質界面之TE波斜向入射

入射波	反射波	透射波
$\vec{E_i} = \hat{a}_y E_{i0} e^{-jk_{1x}x} e^{-jk_{1z}z}$	$\vec{E_r} = \hat{a}_y E_{r0} e^{-jk_{1x}x} e^{+jk_{1z}z}$	$\vec{E_t} = \hat{a}_y E_{t0} e^{-jk_{2x}x} e^{-jk_{2z}z}$
$\vec{H_i} = (-\hat{a}_x \frac{E_{i0}}{\eta_1}\cos\theta_i + \hat{a}_z \frac{E_{i0}}{\eta_1}\sin\theta_i) \cdot e^{-jk_{1x}x} e^{-jk_{1z}z}$	$\vec{H_r} = (\hat{a}_x \frac{E_{r0}}{\eta_1}\cos\theta_i + \hat{a}_z \frac{E_{r0}}{\eta_1}\sin\theta_i) \cdot e^{-jk_{1x}x} e^{+jk_{1z}z}$	$\vec{H_t} = (\hat{a}_x \frac{E_{t0}}{\eta_2}\cos\theta_t + \hat{a}_z \frac{E_{t0}}{\eta_2}\sin\theta_t) \cdot e^{-jk_{2x}x} e^{-jk_{2z}z}$

- 關係式： $1 + \Gamma_{TE} = \tau_{TE}$ ⟶ $\Gamma_{TE} = \dfrac{\frac{\eta_2}{\cos\theta_t} - \frac{\eta_1}{\cos\theta_i}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}}$, $\tau_{TE} = \dfrac{2 \cdot \frac{\eta_2}{\cos\theta_t}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}}$
- 司乃爾定律： $n_1 \sin\theta_1 = n_2 \sin\theta_2$

NO.

DATE / /

4. 介質與介質界面之 TM 波斜向入射

入射波	反射波	透射波
$\vec{E}_i = (\hat{a}_x E_{i0}\cos\theta_i - \hat{a}_z E_{i0}\sin\theta_i)\cdot e^{-jk_{1x}x} e^{-jk_{1z}z}$	$\vec{E}_r = (\hat{a}_x E_{r0}\cos\theta_i + \hat{a}_z E_{r0}\sin\theta_i)\cdot e^{-jk_{1x}x} e^{+jk_{1z}z}$	$\vec{E}_t = (\hat{a}_x E_{t0}\cos\theta_t - \hat{a}_z E_{t0}\sin\theta_t)\cdot e^{-jk_{2x}x} e^{-jk_{2z}z}$
$\vec{H}_i = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-jk_{1x}x} e^{-jk_{1z}z}$	$\vec{H}_r = -\hat{a}_y \frac{E_{r0}}{\eta_1} e^{-jk_{1x}x} e^{+jk_{1z}z}$	$\vec{H}_t = \hat{a}_y \frac{E_{t0}}{\eta_2} e^{-jk_{2x}x} e^{-jk_{2z}z}$

• 關係式： $\cos\theta_i + \Gamma_{TM}\cos\theta_i = \tau_{TM}\cos\theta_t$

$$\Rightarrow \begin{cases} \Gamma_{TM} = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \\[2ex] \tau_{TM} = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \end{cases}$$

• 布魯斯特角： $\theta_i = \theta_B = \tan^{-1}\frac{n_2}{n_1}$

七、相速和群速

1. 相速： $v_p = \frac{\omega}{\beta}$

2. 群速： $v_g = \frac{d\omega}{d\beta}$

3. 相速與群速之關係：

$$v_g = \frac{v_p}{1 - \frac{\omega}{v_p}\cdot\frac{dv_p}{d\omega}}$$

NO.

DATE / /

§ Transmission Lines

一、傳輸線的衰減係數與相位常數

(一) 通式

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (\omega^2LC - RG)\right]}$$

$$\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (\omega^2LC - RG)\right]}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{\frac{1-j\frac{R}{\omega L}}{1-j\frac{G}{\omega L}}}$$

(二) $\omega L \ll R$, $\omega C \ll G$ 低頻大損耗

$$\alpha = \sqrt{RG}, \quad \beta = 0, \quad Z_0 = \sqrt{\frac{R}{G}}$$

(三) $\omega L \gg R$, $\omega C \gg G$ 高頻小損耗(微波)

$$\alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right), \quad \beta \cong \omega\sqrt{LC}$$

$$Z_0 \cong \sqrt{\frac{L}{C}}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} - \frac{G}{\omega C}\right)\right]$$

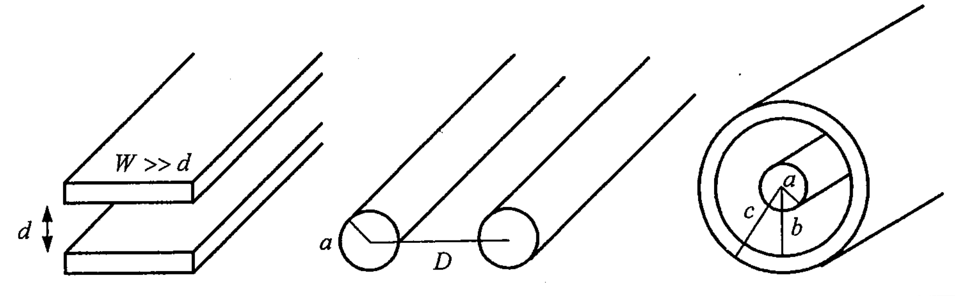
(四) 無損 $R = G = 0$

$$\alpha = 0, \quad \beta = \omega\sqrt{LC}, \quad Z_0 = \sqrt{\frac{L}{C}}, \quad v_p = v_g = \frac{1}{\sqrt{LC}}$$

(五) 無失真傳輸線 $\frac{R}{L} = \frac{G}{C}$

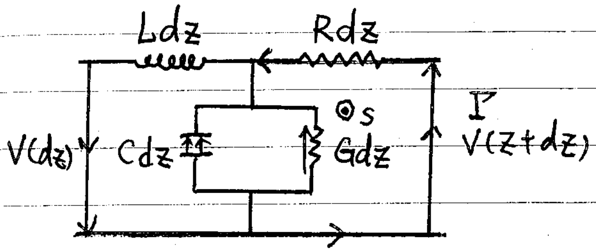
$$\alpha = R\sqrt{\frac{C}{L}}, \quad \beta = \omega\sqrt{LC}, \quad Z_0 = \sqrt{\frac{L}{C}}, \quad v_p = v_g = \frac{1}{\sqrt{LC}}$$

二、傳輸線參數表



R	$\frac{2}{W}\sqrt{\frac{\pi f\mu_c}{\sigma_c}}$	$\frac{1}{\pi a}\sqrt{\frac{\pi f\mu_c}{\sigma_c}}$	$\frac{1}{2\pi}(\frac{1}{a}+\frac{1}{b})\sqrt{\frac{\pi f\mu_c}{\sigma_c}}$
L	$\mu\frac{d}{W}$	$\frac{\mu}{\pi}\ln\left[\frac{D}{2a}+\sqrt{(\frac{D}{2a})^2-1}\right]$ $\frac{\mu}{\pi}\ln(\frac{D}{a})$，if $D>>a$	$\frac{\mu}{2\pi}\ln\frac{b}{a}$
G	$\sigma\frac{W}{d}$	$\frac{\sigma\pi}{\ln\left[\frac{D}{2a}+\sqrt{(\frac{D}{2a})^2-1}\right]}$ $\frac{\sigma\pi}{\ln\frac{D}{a}}$，if $D>>a$	$\frac{\sigma 2\pi}{\ln\frac{b}{a}}$
C	$\varepsilon\frac{W}{d}$	$\frac{\varepsilon\pi}{\ln\left[\frac{D}{2a}+\sqrt{(\frac{D}{2a})^2-1}\right]}$ $\frac{\varepsilon\pi}{\ln\frac{D}{a}}$，if $D>>a$	$\frac{\varepsilon 2\pi}{\ln\frac{b}{a}}$
Z_0	$\eta\frac{d}{W}$	$\frac{\eta}{\pi}\ln\left[\frac{D}{2a}+\sqrt{(\frac{D}{2a})^2-1}\right]$ $\frac{\eta}{\pi}\ln\frac{D}{a}$，if $D>>a$	$\frac{\eta}{2\pi}\ln\frac{b}{a}$

三. 電報方程 → 基本方程.



聯立偏微方:

KVL $\frac{\partial V}{\partial z}+(R+L\frac{\partial}{\partial t})I=0$

KCL $\frac{\partial I}{\partial z}+(C\frac{\partial}{\partial t}+G)V=0$

四. 電壓波 & 電流波

1. 傳輸線上的電壓波與電流波之振幅均為固定值。
2. 輸入端之阻抗為 $Z_{in}=Z_0$, 即輸入阻抗與特性阻抗同。
3. 傳輸線之電压波與电流波不存在相差, 即同相位 (in phase).
4. 駐波比, 定義為电压波振幅的最大值比上最小值。

定義: 电压駐波比 (Voltage) Standing Wave Ratio

$$VSWR \equiv S \underset{\text{Def.}}{\equiv} \frac{|V(z')|_{max}}{|V(z')|_{min}} \underset{\text{公式}}{=} \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

(駐波的成分占行進波的比值)

五、史密司圖(Smith Chart)

$$\begin{cases} \Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} \\ \dfrac{Z_L}{Z_0} = \dfrac{1+\Gamma_L}{1-\Gamma_L} \end{cases} \overset{}{\underset{Z_{in}(z')}{\Longrightarrow}} \begin{cases} \Gamma = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0} \equiv \Gamma_r + j\Gamma_i \\ \dfrac{Z_{in}}{Z_0} = \dfrac{1+\Gamma}{1-\Gamma} \equiv r + jx \quad \text{normulized 輸入阻抗} \end{cases}$$

$$Z_{in} \equiv \frac{V(z')}{I(z')} = Z_0 \frac{1+(\Gamma_L e^{-2\gamma z'})}{1-(\Gamma_L e^{-2\gamma z'})} \quad \begin{matrix}\Gamma(z') \\ \Gamma(z')\end{matrix}$$

$z = \dfrac{Z'}{Z_0}$ 歸一阻抗; $y = YZ_0$ 歸一導納

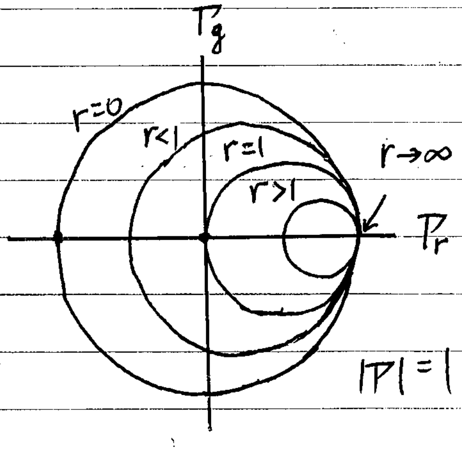
$$\Gamma(z') \equiv \frac{V_-(z')}{V_+(z')} = \Gamma_L e^{-2\gamma z'} = |\Gamma_L| e^{j(\theta_r - 2\beta z')}$$

r圓: $\left(\Gamma_r - \dfrac{r}{r+1}\right)^2 + \Gamma_i^2 = \left(\dfrac{1}{r+1}\right)^2$

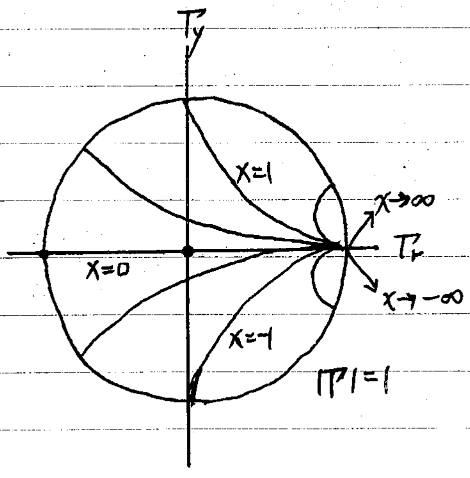
心: $\left(\dfrac{r}{r+1}, 0\right)$ 徑: $\dfrac{1}{r+1}$

x圓: $(\Gamma_r - 1)^2 + \left(\Gamma_i - \dfrac{1}{x}\right)^2 = \left(\dfrac{1}{x}\right)^2$

心: $\left(1, \dfrac{1}{x}\right)$ 徑: $\dfrac{1}{|x|}$



r圓示意圖



x圓示意圖

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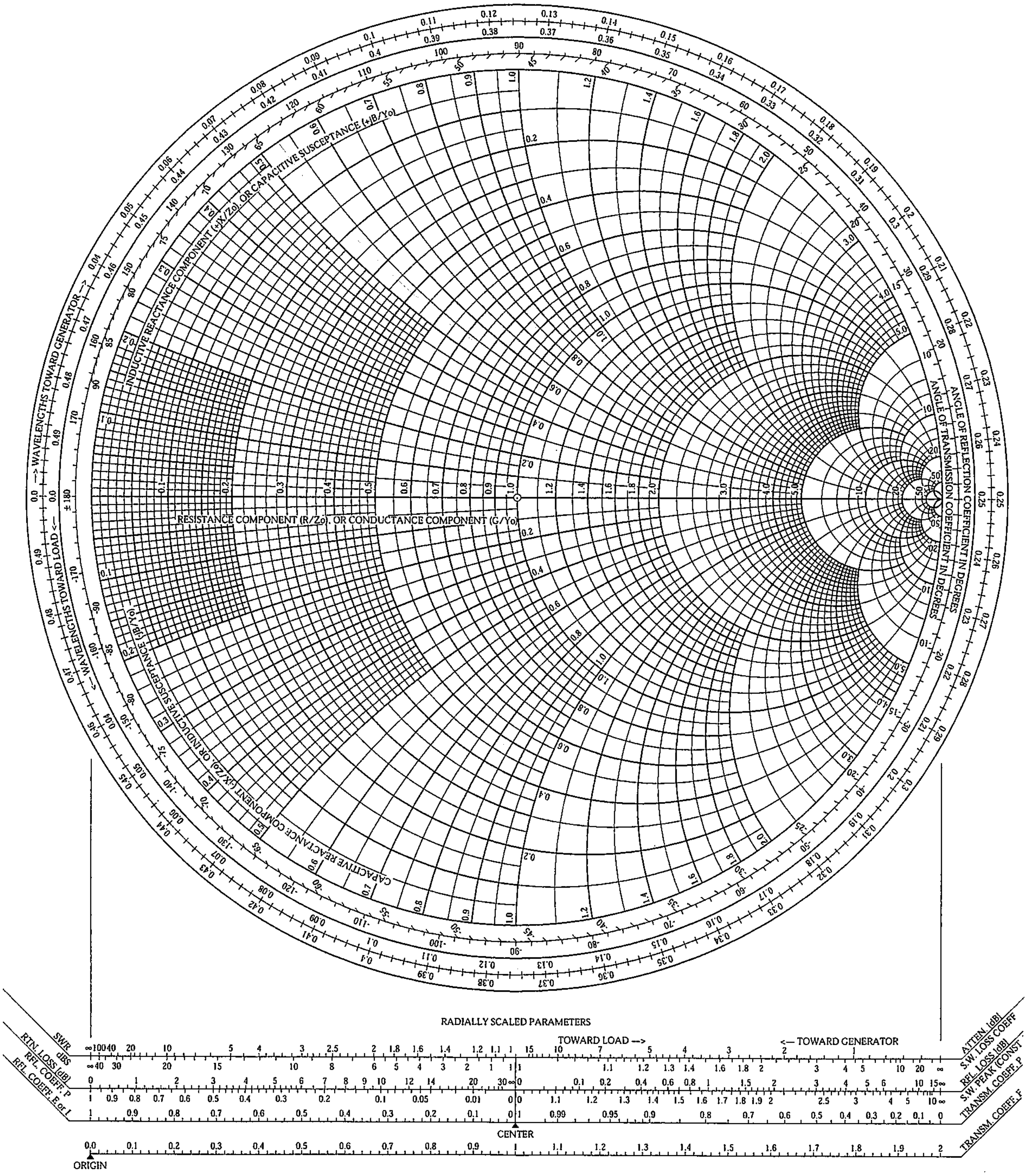
The Complete Smith Chart

Black Magic Design



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DATE / /

波導理論（§波導&共振腔）

一、E_z 與 H_z 法則

無損導波結構	有損導波結構
$H_x=\frac{1}{k_c^2}\left(j\omega\varepsilon\frac{\partial E_z}{\partial y}-j\beta\frac{\partial H_z}{\partial x}\right)$	$H_x=\frac{1}{k_c^2}\left(j\omega\varepsilon\frac{\partial E_z}{\partial y}-r\frac{\partial H_z}{\partial x}\right)$
$H_y=\frac{1}{k_c^2}\left(-j\omega\varepsilon\frac{\partial E_z}{\partial x}-j\beta\frac{\partial H_z}{\partial y}\right)$	$H_y=\frac{1}{k_c^2}\left(-j\omega\varepsilon\frac{\partial E_z}{\partial x}-r\frac{\partial H_z}{\partial y}\right)$
$E_x=\frac{1}{k_c^2}\left(-j\beta\frac{\partial E_z}{\partial x}-j\omega\mu\frac{\partial H_z}{\partial y}\right)$	$E_x=\frac{1}{k_c^2}\left(-r\frac{\partial E_z}{\partial x}-j\omega\mu\frac{\partial H_z}{\partial y}\right)$
$E_y=\frac{1}{k_c^2}\left(-j\beta\frac{\partial E_z}{\partial y}+j\omega\mu\frac{\partial H_z}{\partial x}\right)$	$E_y=\frac{1}{k_c^2}\left(-r\frac{\partial E_z}{\partial y}+j\omega\mu\frac{\partial H_z}{\partial x}\right)$
$k_c^2=k^2-\beta^2$	$k_c^2=k^2+r^2$

二、基本方程&波型特性

1、傳輸線之物理特性：

TEM 阻抗匹配、低頻

low-loss $\alpha=\frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right)$ $R\propto R_s\propto\alpha\sqrt{f}$ $f\uparrow\Rightarrow\alpha\uparrow$ 會有損耗

∴傳輸線不適合傳輸高頻的波

2、基本方程：

TEM波：

$$\underbrace{\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}+k^2\right)\vec{E}(x,y,z)=0}_{\text{Helmholtz's eq.}}\xrightarrow{\frac{\partial}{\partial z}=-j\beta}\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\vec{e}(x,y)=0$$

$$\underbrace{\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}+k^2\right)\vec{H}(x,y,z)=0}_{\text{Helmholtz's eq.}}\xrightarrow{\frac{\partial}{\partial z}=-j\beta}\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\vec{h}(x,y)=0$$

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DATE / /

TE & TM 波:

$$\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}+k^2\right)\vec{E}(x,y,z)=0 \longrightarrow \left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+k_c^2\right)\vec{e}(x,y)=0$$

Helmholtz's eq.

$$\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}+k^2\right)\vec{H}(x,y,z)=0 \longrightarrow \left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+k_c^2\right)\vec{h}(x,y)=0$$

Helmholtz's eq.

三、平行板波導

<1> 平行板傳輸線的 TEM 波

1. 电压波與电流波:

$$V=V_0e^{-jkz}$$

$$I=\frac{V_0}{Z}e^{-jkz}$$

$$Z_0=\eta\cdot\frac{d}{W}$$

2. 电磁波:

$$\vec{E}(z)=\hat{a}_x\frac{V_0}{d}e^{-jkz}$$

$$\vec{H}(z)=\frac{1}{\eta}\hat{a}_z\times\vec{E}=\hat{a}_y\frac{V_0}{\eta d}e^{-jkz}$$

3. 平行板傳輸線也稱為平行板波導,

截止頻率為零 $f_c(TEM)=0$

<2> 平行板波導的 TM 波

1. 場型分佈:

$$\boxed{E_z(y,z) = A_n \sin\frac{n\pi}{d}y \cdot e^{-j\beta z}}$$

$\begin{cases} A_n = const \neq 0 \\ n = 1, 2, 3, \cdots \end{cases}$

$\boxed{H_z = 0}$ (TM 波)

由 E_z-H_z 法則可求:

$$\boxed{H_x = \frac{j\omega\varepsilon}{k_c} A_n \cos\frac{n\pi y}{d} \cdot e^{-j\beta z}}$$

$$\boxed{E_y = \frac{-j\beta}{k_c} A_n \cos\frac{n\pi y}{d} \cdot e^{-j\beta z}}$$

$\boxed{E_x = 0}$

$\boxed{H_y = 0}$

$\Rightarrow$ 綜合成电磁場分佈 $\vec{E}$, $\vec{H}$

2. 導通條件:

$k > k_c$

$$f > f_c = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}} = \frac{n}{2d\sqrt{\mu\varepsilon}}$$

$\lambda < \lambda_c$

3. $\boxed{V_p \cdot v_g = c^2}$

<3> 平行板波導的 TE 波

1. 場型分佈:

$\boxed{E_z = 0}$ (TE 波)

$$\boxed{H_z(y,z) = A_n \cos\frac{n\pi}{d}y \, e^{-j\beta z}}$$

$\begin{cases} A_n = const \neq 0 \\ n = 1, 2, 3, \cdots \end{cases}$

由 E_z-H_z 法則可求:

$$\boxed{E_x = \frac{j\omega\mu}{k_c} A_n \sin\frac{n\pi}{d}y \cdot e^{-j\beta z}}, \quad n = 1, 2, 3, \cdots$$

$$\boxed{H_y = \frac{j\beta}{k_c} A_n \sin\frac{n\pi}{d}y \cdot e^{-j\beta z}}, \quad n = 1, 2, 3, \cdots$$

$\boxed{E_y = 0}$

$\boxed{H_x = 0}$

綜合成电磁場分佈 $\vec{E}$、$\vec{H}$

2. 導通條件同 TM 波

四、矩形金屬波導

<1> 矩形金屬波導中的 TM 波

1. 場型:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)E_z(x,y,z)=0 \qquad \text{Helmholtz's eq.}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)e_z(x,y)=0 \qquad \text{D.E.}$$

$$k_c=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$\beta=\sqrt{k^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}$$

2. 導通條件:

$$k>k_c$$

$$f>f_c=\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$\lambda<\lambda_c=\frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

3. 主模: $TM \rightarrow TM_{11}$

$$f_c^{11}=\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

4. $$\boxed{E_z(x,y,z)=A_{11}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}e^{-j\beta z}} \qquad H_z=0$$

由 E_z、H_z 法則 $\Longrightarrow \vec{E},\vec{H}$

(2) 矩形金屬波導中的 TE 波

1. 場型：

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\right)H_z(x,y,z)=0$$ Helmholtz's eq.

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_c^2\right)h_z(x,y)=0$$ D.E.

$$k_c=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$\beta=\sqrt{k^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}$$

2. 導通條件：

$$k>k_c$$

$$f>f_c=\frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$\lambda<\lambda_c=\frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

3. 主模： TE $\begin{cases} a>b & TE_{10} \\ a<b & TE_{01}\end{cases}$

由截止頻率公式，取 $m=1, n=0$ 或 $m=0, n=1$ 可得

$$\left.\begin{aligned} f_c(TE_{10})&=\frac{1}{2\sqrt{\mu\epsilon}}\frac{1}{a} \\ f_c(TE_{01})&=\frac{1}{2\sqrt{\mu\epsilon}}\frac{1}{b}\end{aligned}\right. \Rightarrow \begin{cases} a>b, \text{ 主模為 } TE_{10} \\ a<b, \text{ 主模為 } TE_{01}\end{cases}$$

4. $\boxed{H_z(x,y,z)=A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}}$ $\quad E_z=0$

由 E_z-H_z 法則 $\Rightarrow \vec{E},\vec{H}$

$$\begin{cases} E_x = \frac{1}{k_c^2}\left[-j\omega\mu\frac{\partial H_z}{\partial y}\right] = \frac{j\omega\mu}{k_c^2}\frac{n\pi}{b}A_{mn}\cos\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\,e^{-j\beta z} \\ E_y = \frac{1}{k_c^2}\left[j\omega\mu\frac{\partial H_z}{\partial x}\right] = \frac{-j\omega\mu}{k_c^2}\frac{m\pi}{a}A_{mn}\sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\,e^{-j\beta z} \\ H_x = \frac{1}{k_c^2}\left[-j\beta\frac{\partial H_z}{\partial x}\right] = \frac{j\beta}{k_c^2}\frac{m\pi}{a}A_{mn}\sin\frac{m\pi}{a}x\cos\frac{n\pi}{b}y\,e^{-j\beta z} \\ H_y = \frac{1}{k_c^2}\left[-j\beta\frac{\partial H_z}{\partial y}\right] = \frac{j\beta}{k_c^2}\frac{n\pi}{b}A_{mn}\cos\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\,e^{-j\beta z} \end{cases}$$

五、圓形金屬波導

1. $k_c^2 = k^2 - \beta^2$
2. TE_{nm} 模態的求解 $\Rightarrow J_n'(k_c a) = 0$
3. TM_{nm} 模態的求解 $\Rightarrow J_n(k_c a) = 0$

$$\boxed{E_z = (C_1 \sin n\phi + C_2 \cos n\phi)\, J_n(k_c r)\, e^{-j\beta z}}$$

六、共振腔

<1> TM_{mnp} to z

1. $E_z = A_{mnp}\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y\,\cos\frac{p\pi}{d}z$
2. 共振波數 $k_{mnp} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$ $\begin{cases} A_{mnp} = const. \neq 0 \\ m, n = 1, 2, 3, \cdots \\ p = 0, 1, 2, \cdots \end{cases}$

   共振頻率 $f_{mnp} = \frac{k_{mnp}}{2\pi\sqrt{\mu\epsilon}}$
3. 主模： $TM_{110} \longrightarrow f_{110} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$

<2> TE_{mnp} to z

1. $H_z = A_{mnp}\cos\frac{m\pi}{a}x\,\cos\frac{n\pi}{b}y\,\sin\frac{p\pi}{d}z$
2. 共振波數 k_{mnp} 及共振頻率 f_{mnp} 同 TM_{mnp}

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DATE / /

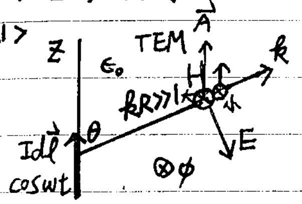
3. 主模:

$$\begin{cases} a>b \text{ ，主模為 } TE_{101} & f_{101}=\dfrac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\dfrac{1}{a^2}+\dfrac{1}{d^2}} \\ a<b \text{ ，主模為 } TE_{011} & f_{011}=\dfrac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\dfrac{1}{b^2}+\dfrac{1}{d^2}} \end{cases}$$

天線 (2007 Note)

一、短天線 (Hertzian Dipole, HD)

<1>



1. 遠場 ≙ $kR \gg 1$
2. $\vec{H} = \hat{a}_\phi \frac{1}{4\pi} I d\ell \frac{e^{-jkR}}{R} jk \sin\theta$
3. $\vec{E} = \hat{a}_\theta \eta \frac{1}{4\pi} I d\ell \frac{e^{-jkR}}{R} jk \sin\theta$

<2> 天線的解題步驟:

$$\vec{J} \xrightarrow{①} \vec{A} \xrightarrow{②} \vec{H} \xrightarrow{③} \vec{E}$$

① $\rightarrow$ $\vec{A} = \int_V \frac{\mu_0}{4\pi} \frac{\vec{J} dV}{R} e^{-jkR}$

② $\xrightarrow[\vec{B}=\mu_0\vec{H}=\nabla\times\vec{A}]{}$ $\vec{H} = \frac{1}{\mu_0} \nabla\times\vec{A}$

③ $\xrightarrow[\nabla\times\vec{H}=j\omega\varepsilon_0\vec{E}]{}$ $\vec{E} = \frac{1}{j\omega\varepsilon_0} \nabla\times\vec{H}$

球 $\propto \frac{e^{-jkR}}{R}$

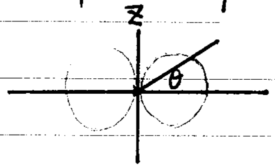
平 $\propto e^{-jkR}$

柱 $\propto \frac{e^{-jkR}}{\sqrt{r}}$

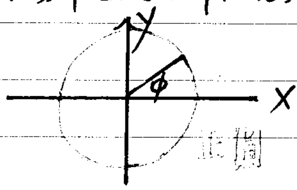
二. 名詞定義:

1. 輻射場型: 將天線所輻射的电場对θ和φ作圖。

2. 电場平面(E-plane): E Plane ≜ $|E|$ vs. θ 作圖 $|E| \propto |\sin\theta|$



3. 磁場平面(H-plane): H Plane ≜ $|E|$ vs. φ 作圖 $|E|$ indep. φ



4. 波印亭相量: $\vec{P}_{av} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*) = \hat{a}_R \frac{1}{2} \eta \frac{1}{16\pi^2} I^2 dl^2 k^2 \sin^2\theta \frac{1}{R^2}$

$$= \boxed{\hat{a}_R \frac{\eta}{32\pi^2} \frac{I^2 dl^2}{R^2} k^2 \sin^2\theta}$$

5. 輻射強度: $U(\theta,\phi) \triangleq P_{av}(R,\theta,\phi) R^2 = \boxed{\frac{\eta}{32\pi^2} I^2 dl^2 k^2 \sin^2\theta}$

6. 輻射功率: $P_r = \oint_s (\vec{E} \times \vec{H}) \cdot d\vec{S}$

$$\boxed{P_r = \frac{\eta}{12} \frac{I^2 dl^2}{\pi} k^2}$$

7. 輻射电阻(R_r): $P_r \triangleq \frac{1}{2} I^2 R_r \Rightarrow \boxed{R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$

8. 方向增益(Direction gain, G_D):

$G_D \triangleq \frac{U(\theta,\phi)}{(P_r/4\pi)}$ ← 平均輻射功率

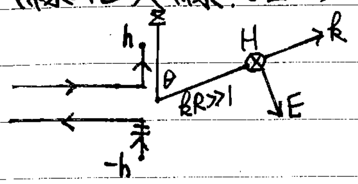
HD.

$\boxed{G_D = \frac{3}{2}\sin^2\theta}$

9. 方向性(Directivity, D):

$D \triangleq Max(G_D)$ $\boxed{G_D = \frac{3}{2}}$ for HD

三、線性天線 (LA)



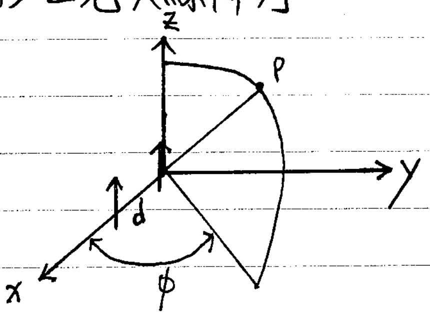
$$\vec{E} = \hat{a}_\theta\, j\, F(\theta)\, \frac{e^{-jkR}}{R}$$

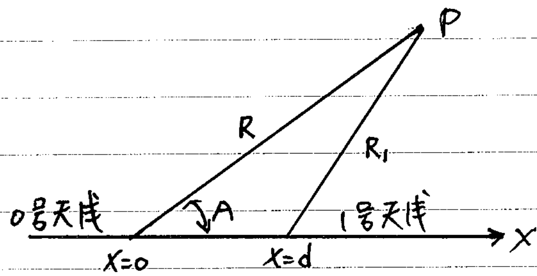
$$F = \frac{\cos(kh\cos\theta) - \cos(kh)}{\sin\theta}\, 60\, I_m$$

1. $I(z) = I_m \sin k(h - |z|)$
2. $\because$ TEM $\therefore \vec{H} = \frac{1}{\eta}\hat{a}_R \times \vec{E} = \hat{a}_\phi \frac{jF}{\eta} \frac{e^{-jkR}}{R}$
3. $\theta = 0 \Rightarrow H = E = 0$ HD=0 , $LA = \int HD = 0$

四、天線陣列:

<1> 二元天線陣列:





$\vec{E} = \vec{E}_0 + \vec{E}_d = \vec{E}_0 \underbrace{(1 + e^{j\varphi})}_{\text{AF 陣列因子}}$

EF 基本因子

$\varphi = \alpha + kd\sin\theta\cos\phi$

$I_d = I_0 e^{j\alpha}$

$\vec{E} = \hat{a}_\theta E_m(\theta,\phi) \frac{e^{-jkR}}{R}$

1. $NAF \triangleq \frac{|AF|}{2} = \left|\cos\frac{\varphi}{2}\right|$, $AF \triangleq 1 + e^{j\varphi}$
2. $\vec{E} = \hat{a}_\theta E_m(\theta,\phi) \frac{e^{-jkR}}{R} e^{j\frac{\varphi}{2}}\, 2\cdot\cos\frac{\varphi}{2}$

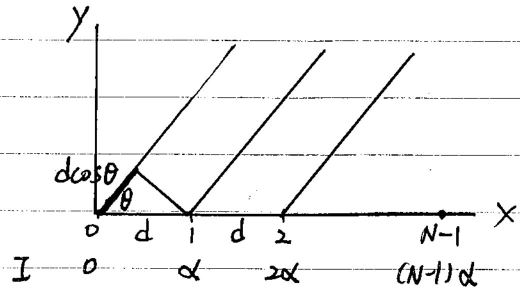
   $\vec{H} = \hat{a}_\phi \frac{E_m(\theta,\phi)}{\eta} \frac{e^{-jkR}}{R} e^{j\frac{\varphi}{2}}\, 2\cdot\cos\frac{\varphi}{2}$

   $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ (\Omega)$

NO.

DATE / /

<2> N元天線陣列:



1. $AF = 1 + e^{j\varphi} + e^{j2\varphi} + \cdots + e^{j(N-1)\varphi}$

$\varphi = \alpha + k d\cos\theta$

$$AF = \frac{1-e^{jN\varphi}}{1-e^{j\varphi}} = e^{j\frac{(N-1)\varphi}{2}} \frac{\sin\frac{N\varphi}{2}}{\sin\frac{\varphi}{2}} \text{※}$$

2. $$NAF \triangleq \frac{|AF|}{N} = \frac{1}{N}\left|\frac{\sin\frac{N\varphi}{2}}{\sin\frac{\varphi}{2}}\right| \text{※}$$