NO.

DATE 2017 / 10 / 01

電磁理論 (Note)

§ CH 1. EM fields

一、Maxwell Equation and Continuty Equation :

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (Faraday's law) $\xrightarrow[\nabla\cdot]{}$ $\nabla\cdot\vec{B}=0$ 散度為0→沿向量方向無变化

$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J}_i$ (Ampere's law) $\xrightarrow[\nabla\cdot]{}$ $\nabla\cdot\vec{D}=\rho_{ev}$ 散度不為0

电流造成磁場

$\nabla\cdot\vec{J} + \frac{\partial\rho_{ev}}{\partial t} = 0$ (Continuty equation) →有獨立存在的电荷

电荷量的時間变动相當於电流

$\nabla\cdot\vec{D} = \rho_{ev}$ 电荷造成电場

$\nabla\cdot\vec{B} = 0$ 沒有獨立磁荷

二、Constitutive Relations

$\vec{D}(\vec{r},t) = \varepsilon_0\vec{E}(\vec{r},t)$

$\vec{B}(\vec{r},t) = \mu_0\vec{H}(\vec{r},t)$

$\mu_0 = 4\pi\times10^{-7}$ (H/m)

$\varepsilon_0 = 8.854\times10^{-12}$ (F/m)

$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \cong 2.998\times10^{8}$ (m/s)

$\varepsilon_0 = \frac{1}{\mu_0 c^2} = \frac{10^{-9}}{36\pi}$ (F/m)

$c = 3\times10^{8}$ (m/s)

三、Magnetic Charges and Currents

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} - \vec{M}_i$

$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} + \vec{J}_i + \vec{J}_c$

$\boxed{\nabla\cdot\vec{M}_i + \frac{\partial\rho_{mv}}{\partial t} = 0}$

$\boxed{\nabla\cdot\vec{J}_{ic} + \frac{\partial\rho_{ev}}{\partial t} = 0}$

$\nabla\cdot\nabla\times E = -\frac{\partial}{\partial t}\nabla\cdot\vec{B} - \nabla\cdot\vec{M}_i$

$= -\frac{\partial}{\partial t}\nabla\cdot\vec{B} + \frac{\partial\rho_{mv}}{\partial t} = 0$ $\Rightarrow$ $\boxed{\nabla\cdot\vec{B} = \rho_{mv}}$

NO.

DATE / /

四、Maxwell Equation in Integral Form

$$\oint_C \vec{E}\cdot d\vec{l} = -\iint_S \vec{M}_i \cdot d\vec{s} - \frac{d}{dt}\iint_S \vec{B}\cdot d\vec{s} \quad \text{Faraday's Law}$$

电压　穿出的磁流量　穿出的磁通量時变率

$$\oint_C \vec{H}\cdot d\vec{l} = \iint_S \vec{J}_{ic}\cdot d\vec{s} + \frac{d}{dt}\iint_S \vec{D}\cdot d\vec{s} \quad \text{Ampere's Law}$$

电流　穿出的电流量　穿出的电通量時变率

$$\oint_S \vec{D}\cdot d\vec{s} = Q_e$$

穿出的电通量

$$\oint_S \vec{B}\cdot d\vec{s} = Q_{mv}$$

五、Circuit - Feild Relations

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_S \vec{B} d\vec{s} = -\frac{d}{dt}\psi_m$$

$$\Rightarrow \sum_{n=1}^{N} V_n = -\frac{d}{dt}L_s i = -L_s\frac{di}{dt} \quad \boxed{\psi_m = L_s i} \quad L_s: \text{stray inductance}$$

$$L_s \simeq 0 \text{ in low frequency} \Rightarrow \sum_{n=1}^{N} V_n = 0$$

$$\oint_S \vec{J}_{ic} d\vec{s} = -\frac{dQ_e}{dt}$$

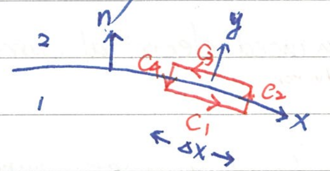
$$\Rightarrow \sum_{n=1}^{N} i_n = -\frac{d}{dt}C_s V = -C_s\frac{dV}{dt} \quad \boxed{Q_e = C_s V} \quad C_s: \text{stray capacitiance}$$

$$C_s \simeq 0 \text{ in low frequency} \Rightarrow \sum_{n=1}^{N} i_n = 0$$

$\vec{J}(r,t) = \sigma_0 \vec{E}(r,t)$ 可推得 $V = \frac{L}{\Delta S \sigma_0} I$

$$\because R = \frac{L}{\Delta S \sigma_0} \quad \therefore V = R\cdot I$$

六. Boundary Conditions



电通量趨近於0

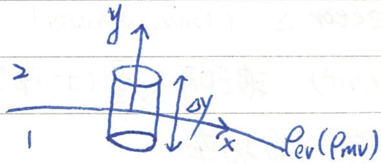
由 Ampere's Law, with $\iint_S \vec{D}\cdot d\vec{s} \to 0$

$\Rightarrow$ if $\vec{J}_{is}=0$ $\Rightarrow$ $\boxed{\hat{n}\times(\vec{H}_2-\vec{H}_1)=0}$

若表面电流為0, 則界面兩側的 tangential magnetic field intensity 相同

同理 (Faraday's Law) $\Rightarrow$ $\boxed{\hat{n}\times(\vec{E}_2-\vec{E}_1)=-\vec{M}_{is}}$

若表面磁流為0, 則界面兩側的 tangential electric field intensity 相同



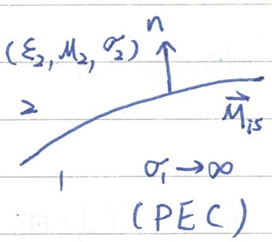
$\rho_{es}=\lim_{\Delta y\to 0}\rho_{ev}\Delta y$ (wb/m²) 當圓筒压扁→只剩下表面电荷

$\Rightarrow$ $\boxed{\hat{n}\cdot(\vec{D}_2-\vec{D}_1)=\rho_{es}}$ 表面电荷密度相當於界面兩側的 normal electric flux density difference

$\rho_{ms}=\lim_{\Delta y\to 0}\rho_{mv}\Delta y$ (wb/m²) 當圓筒压扁→只剩下表面磁荷

$\Rightarrow$ $\boxed{\hat{n}\cdot(\vec{B}_2-\vec{B}_1)=\rho_{ms}}$ 表面磁荷密度相當於界面兩側的 normal magnetic flux density difference

- On the PEC boundary

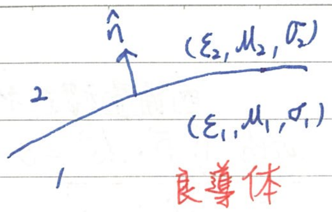


$\hat{n}\times\vec{E}_2=-M_{is}^{induced}$ 金屬表面也無 tangential 电場
↳產生等效磁流

$\hat{n}\times\vec{H}_2=\vec{J}_s^{impressed}$ 金屬表面有电流就會有 tangential 磁場

$\hat{n}\cdot\vec{B}_1=\rho_{ms}^{impressed}$ 金屬表面無 normal 磁通量/磁場
↳產生等效磁荷

$\hat{n}\cdot\vec{D}_2=\rho_{es}^{impressed}$ 金屬表面有电荷就會有 normal 电通量/电場



良導体

(σ_1 is large, finite)

$\hat{n}\times\hat{n}\times\vec{E}_2 = -Z_s\,\hat{n}\times\vec{H}_2 \Rightarrow \boxed{\hat{n}\times\vec{M}_s = Z_s\vec{J}_s}$

$\hat{n}\times(\vec{H}_2-\vec{H}_1)=\vec{J}_s$

$\vec{J}_s = \hat{n}\times\vec{H} \rightarrow$ induced electrical current

Good Conductor

$\hat{n}\times(\vec{E}_2-\cancel{\vec{E}_1}) = -\vec{M}_s$

$\vec{M}_s = -\hat{n}\times\vec{E}_2 \rightarrow$ induced magnetic current

結論：阻抗表面上有 tangentid 电場 → 有等效磁流

七. Power and Energy

$$\nabla\times\vec{E} = -\vec{M}_i - \frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{H} = \underset{\text{impressed}}{\vec{J}_i} + \underset{\text{induced}}{\vec{J}_c} + \frac{\partial\vec{D}}{\partial t} \Rightarrow$$

1. Poynting vector $\vec{S}$ (Power density)
   $\vec{E}\times\vec{H} = \vec{S}$ (W/m^2) 波印亭向量(功率密度)
2. P_e：離開封閉面的功率
   $P_e = \oiint_S \vec{S}\cdot d\vec{s}$ (W)
3. P_s：進入封閉面的功率
   $P_s = -\iiint_V (\vec{H}\cdot\vec{M}_i + \vec{E}\cdot\vec{J}_i)\,dv$
4. P_d：在封閉面內消耗的功率
   $P_d = \iiint_V \vec{E}\cdot\vec{J}_c\,dv = \iiint_V \sigma\vec{E}^2\,dv$
5. 以磁場形式儲存的能量密度：
   $W_m = \frac{1}{2}\mu\vec{H}^2$ (magnetic energy density (J/m^3))
6. 以电場形式儲存的能量密度：
   $W_e = \frac{1}{2}\varepsilon\vec{E}^2$ (electric energy density (J/m^3))

$$\Rightarrow \boxed{P_e + \frac{\partial}{\partial t}\iiint_V (W_e + W_m)\,dv + P_d = P_s}$$

Conservation of energy 能量守恆

NO.

DATE / /

八、Time Harmonic Form

1. 波印亭向量：$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}^*$ (W/m^2)

2. 離開封閉面的功率：$P_e=\oint_S \vec{S}\cdot d\vec{s}$

3. 進入封閉面的功率：$P_s=\iiint_V\left(\frac{1}{2}\vec{H}^*\cdot\vec{M}_i+\frac{1}{2}\vec{E}\cdot\vec{J}_i^*\right)dv$

4. 在封閉面內消耗的功率：$P_d=\iiint_V \frac{1}{2}\sigma\vec{E}\cdot\vec{E}^*dv$

5. 以磁場形式儲存的能量密度：$W_m=\iiint_V \frac{1}{4}\mu\vec{H}\cdot\vec{H}^*dv$

6. 以电場形式儲存的能量密度：$W_e=\iiint_V \frac{1}{4}\varepsilon\vec{E}\cdot\vec{E}^*dv$

$\Rightarrow \boxed{P_s=P_e+P_d+2j\omega(W_m-W_e)}$ 能量守恆

NO.

DATE / /

§CH2. EM Properties

一、Dielectrics, polarization, and permittivity

1、Dielectrics (介电材料)

$\vec{p} = Q\vec{l}$ (C·m) $\longrightarrow d\vec{p}_i = Q\vec{l}_i$

电偶極矩正比於电荷量與位移量，
某些材質的分子結構有自發的电偶極矩，
不過因方向散亂而抵銷，巨观的电偶極矩=0。

2. $\vec{P} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_{i=1}^{N_e \Delta V} d\vec{p}_i$ (C/m^2)

稱為电極化向量

3. $\vec{P} = \frac{1}{\Delta V} N_e \Delta V Q \vec{l}_{ave} = N_e Q \vec{l}_{ave}$

4. 在介电材料中 $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = (\varepsilon_0 + \varepsilon_0 \chi_e)\vec{E}$

$\Rightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E}$ 其中 χ_e：电極化率

ε_s：Static permittivity (F/m) $\rightarrow \boxed{\varepsilon_{rs} = \varepsilon_s / \varepsilon_0 = 1 + \chi_e}$

二、Magnetic Materials

1、磁偶極等同电流迴圈：
Magnetic dipole moment

$d\vec{m}_i = \hat{n}\, ds\, I$ 磁偶極矩=电流迴圈面積*电流
磁偶極矩方向與外加磁場方向平行

NO.

DATE / /

2、磁偶極矩：$\vec{m}=IA\,\hat{a}_n$

3、diamagnetism & paramagnetic 材料
抗磁性 順磁性

抗磁性是材料磁性的一種。當材料垂直於不均勻磁場放置時，磁場會從材料內部被部分地排斥出來。法拉第將它定名為抗磁性。所有物質都具有抗磁性，然而，某些材料的抗磁性或者被弱的磁吸引(順磁性)、或者被很強的磁吸引(鐵磁性)所掩蓋。

4. $\vec{M}=\lim_{\Delta V\to 0}\frac{1}{\Delta V}\sum_{i=1}^{N_m\cdot\Delta V} d\vec{m}_i \quad (N_m)$

稱為 磁極化向量

5. $\vec{M}=\hat{n}N_m(Ids)_{ave}$ $\qquad \vec{M}=\chi_m\cdot\vec{H}$ $\quad \chi_m$：磁極化率

6. 在材料中 $\vec{B}=\mu_0\vec{H}+\mu_0\cdot\vec{M}$

$=\mu_0(1+\chi_m)\vec{H}=\mu_s\vec{H}$ $\quad \mu_s$：Static permeability

$\rightarrow \boxed{\mu_{sr}=\mu_s/\mu_0=1+\chi_m}$

7. $\vec{B}_a=\mu_0 J_{s0}\,\hat{a}_y\times\hat{a}_z$ 沒有介磁材料的感應磁場

介磁材料中沒有淨电流，但介磁材料表面有电流
稱 磁化表面电流

NO.

DATE / /

三、Conductor and Conductivity (導电率)

1. 电槳: $\boxed{\vec{J} = \rho_{ev}^{-}\vec{v}^{-} + \rho_{ev}^{+}\vec{v}^{+}}$

2. 導体: $\boxed{\vec{J} = \rho_{ev}^{-}\vec{v_e} = -\rho_{ev}^{-}\mu_e\vec{E} = \sigma_s\vec{E}}$

3. 半導体: $\boxed{\vec{J} = \rho_{vv}^{-}\vec{v_e} + \rho_{hv}^{+}\vec{v_h}^{+} = \sigma_s\vec{E}}$

Holes

純半導体中導电率小，參雜質可提升導电率

四、Complex Permittivity

1. 討論 EMW 在材料中耗損的物理机制

$$\frac{\partial^2 l}{\partial t^2} + r\frac{\partial l}{\partial t} + \omega_0^2 l = \frac{Q}{m}E(t)$$

代表耗損

by $E(t) = E_0 e^{j\omega x}$

$l = l_0 e^{j\omega x}$

$$\Rightarrow (-\omega^2 + j\omega r + \omega_0^2)\, l_0 = \frac{Q}{m}E_0$$

$$\Rightarrow l_0 = \frac{QE}{m(\omega_0^2 - \omega + j\omega r)} \quad \Rightarrow l = \frac{QEe^{j\omega x}}{m(\omega_0^2 - \omega + j\omega r)}$$

$$\vec{P} = N_e Q(\vec{l_{av}}) = \frac{N_e \frac{Q^2}{m} E e^{j\omega x}}{\omega_0^2 - \omega + j\omega r}$$

2. Good dielectric: $\frac{|J_{cel}|}{|J_{del}|} \ll 1$ 大部分是位移电流

Good conductor: $\frac{|J_{cel}|}{|J_{del}|} \gg 1$ 大部分是傳導电流

NO.

DATE / /

3. $\nabla\times\vec{E} = -\vec{M}_i - j\omega\mu'\vec{H} - j\omega\mu''\vec{H} = -\vec{M}_i - \vec{M}_d - \vec{M}_c$

位移磁流 傳導磁流

$\tan\delta_m = \frac{\mu''}{\mu'}$ 稱磁損失正切

五、Anisotropic Materials：

$\vec{D} = \bar{\bar{\varepsilon}}\vec{E}$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$\vec{B} = \bar{\bar{\mu}}\vec{H}$

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix}$$

$\vec{D} = \bar{\bar{\varepsilon}}\vec{E} + \bar{\bar{\mu}}\vec{H}$ for more complicated material

NO.

DATE / /

§CH3. Wave Equation Solutions

一、Wave Equation Solutions：

由 $\begin{cases} \nabla\times\vec{E} = -\vec{M}_i - j\omega\mu\vec{H} & \text{Faraday's Law} \\ \nabla\times\vec{H} = \vec{J}_i + \sigma_e\vec{E} + j\omega\varepsilon'\vec{E} & \text{Ampere's Law} \end{cases}$

及 $\varepsilon_e = \varepsilon_e' - j\varepsilon_e'' = \varepsilon' - j\frac{\sigma_e}{\omega}$ 等效複數介电係數

可推得 ⇒ (1) Resulting E Field：

$$\boxed{\underbrace{\nabla\times\left(\frac{\nabla\times\vec{E}}{\mu}\right) - \omega^2\varepsilon_e\vec{E}}_{\text{2nd order of }\vec{E}} = \underbrace{-\nabla\times\left(\frac{\vec{M}_i}{\mu}\right) - j\omega\vec{J}_i}_{\text{Unknow source}}}$$

(2) Resulting H Field：

$$\boxed{\underbrace{\nabla\times\left(\frac{\nabla\times\vec{H}}{\varepsilon_e}\right) - \omega^2\mu\vec{H}}_{\text{2nd order of }\vec{H}} = \underbrace{\nabla\times\left(\frac{\vec{J}_i}{\varepsilon_e}\right) - j\omega\vec{M}_i}_{\text{Unknow source}}}$$

二、Homogeneous Materials：

(1) ⇒ $\nabla\times\nabla\times\vec{E} - \omega^2\mu\varepsilon_e\vec{E} = -\nabla\times\vec{M}_i - j\omega\mu\vec{J}_i$

(2) ⇒ $\nabla\times\nabla\times\vec{H} - \omega^2\mu\varepsilon_e\vec{H} = \nabla\times\vec{J}_i - j\omega\varepsilon_e\vec{M}_i$

$r = \sqrt{-\omega^2\mu\varepsilon_e} = \alpha + j\beta$ 傳播常數

$$\Rightarrow \boxed{-\nabla^2\vec{E} - r^2\vec{E} = -\nabla\times\vec{M}_i - j\omega\mu\vec{J}_i - \frac{\nabla\rho_{ev}}{\varepsilon}}$$

$$\boxed{\nabla^2\vec{H} - r^2\vec{H} = -\nabla\times\vec{J}_i + j\omega\varepsilon_e\vec{M}_i + \frac{\nabla\rho_{mv}}{\mu}}$$

空間中磁場分布变化有电流的空間变动、磁流的時間变动，磁荷的空間变化有関

NO.

DATE / /

三、Homogeneous Source-Free Media

$$\begin{cases} \nabla^2\vec{E} - \gamma^2\vec{E} = 0 & \nabla\cdot\vec{E} = 0 \\ \nabla^2\vec{H} - \gamma^2\vec{H} = 0 & \nabla\cdot\vec{H} = 0 \end{cases}$$ in Source-Free Media

定义：$(\nabla^2 - \gamma^2)\psi = 0$，其中 $\psi(\vec{r}) = f(x)g(y)h(z)$

由变数分離法 $\Rightarrow$ $\boxed{\frac{f''}{f} + \frac{g''}{g} + \frac{h''}{h} - \gamma^2 = 0}$ $\gamma^2 = \gamma_x^2 + \gamma_y^2 + \gamma_z^2$

解得 $f(x) = A_1 e^{-\gamma_x x} + B_1 e^{\gamma_x x}$ $f(x) = C_1\cosh(\gamma_x x) + D_1\sinh(\gamma_x x)$

$\uparrow$ 行波 $\uparrow$ 駐波

$\gamma_x = \alpha_x + j\beta_x$

A_1, B_1, C_1, D_1 找到ABCD就可以從一般解變成特定解(答案)

四、Wave Equation Solutions in Cylindrical Coordinates.

$$\boxed{\rho^2\frac{d^2 f}{d\rho^2} + \rho\frac{df}{d\rho} + [(\rho\gamma_\rho)^2 - m^2]f = 0}$$ 2nd order Bessel differential equation (ρ方向)

1. as $\gamma_\rho\rho \to 0$ 纜偶用導波管的模態

$m>0$ $J_m(\gamma_\rho\rho) \sim \frac{\left(\frac{\gamma_\rho\rho}{2}\right)^m}{m!}$ $Y_m(\gamma_\rho\rho) \sim -\frac{(m-1)!}{\pi} \times \left(\frac{2}{\gamma_\rho\rho}\right)^m$

$$\begin{cases} J_m(\gamma_\rho\rho) \sim \sqrt{\frac{2}{\pi\gamma_\rho\rho}}\cos\left(\gamma_\rho\rho - \frac{m\pi}{2} - \frac{\pi}{4}\right) & \text{as } \gamma_\rho\rho \to \infty \\ Y_m(\gamma_\rho\rho) \sim \sqrt{\frac{2}{\pi\gamma_\rho\rho}}\sin\left(\gamma_\rho\rho - \frac{m\pi}{2} - \frac{\pi}{4}\right) & \text{as } \gamma_\rho\rho \to \infty \end{cases}$$

$$\boxed{f = A_1 J_m(\gamma_\rho\rho) + B_1 Y_m(\gamma_\rho\rho)}$$ ρ方向

NO.

DATE / /

2. $\boxed{f = C_1 H_m^{(1)}(\gamma_\rho \rho) + D_1 H_m^{(2)}(\gamma_\rho \rho)}$ 行進波之Hankel function

$H_m^{(1)} = J_m(\gamma_\rho \rho) + jY_m(\gamma_\rho \rho) \longrightarrow H_m^{(1)}(\gamma_\rho \rho) = \sqrt{\frac{2}{\pi \gamma_\rho \rho}}\, e^{+j(\gamma_\rho \rho - m\frac{\pi}{2} - \frac{\pi}{4})}$

$H_m^{(2)} = J_m(\gamma_\rho \rho) - jY_m(\gamma_\rho \rho) \longrightarrow H_m^{(2)}(\gamma_\rho \rho) = \sqrt{\frac{2}{\pi \gamma_\rho \rho}}\, e^{-j(\gamma_\rho \rho - m\frac{\pi}{2} - \frac{\pi}{4})}$

$\therefore$ As $\gamma_\rho \rho \to \infty$ $H_m^{(1)} \to$ 往 $-\rho$ 方向行進波

$H_m^{(2)} \to$ 往 $+\rho$ 方向行進波

3. 同軸纜線中的电場分布解:

Coaxial Cable:

$\psi = f(\rho) g(\phi) h(z) = E_z$

$= [A_1 J_m(\gamma_\rho \rho) + B_1 Y_m(\gamma_\rho \rho)][C_2 \cos m\phi + D_2 \sin m\phi][A_3 e^{-\gamma_z z} + B_3 e^{\gamma_z z}]$

(ρ方向駐波) (ϕ方向駐波) (z方向行波)

NO.

DATE / /

§CH4 Wave Properties

一、Uniform Plane Waves

- In lossless medium : $(\nabla^2-\gamma^2)\psi=0$

$(\nabla^2+\beta^2)\psi=0$ $\beta=\omega\sqrt{\mu\epsilon}$ 波動方程式

$\boxed{\psi(\vec{r})=A(\vec{r})e^{j\phi(\vec{r})}}$ $\Rightarrow$ Real Wave $=Re(\psi e^{j\omega t})=A(\vec{r})\cos(\omega t+\phi(\vec{r}))$

波动方程式与其解之關係

1. 定義：波的等相位面，又稱為波前 $\phi(\vec{r})=constant.$
($\omega t+\phi(\vec{r})=constant$))

2. 性質：等相位面上的場強相同

3. $\boxed{\vec{\beta}=-\nabla\phi}$ 梯度方向為最大變化率方向

4. $\boxed{\beta_x=-\frac{\partial\phi}{\partial x},\ \beta_y=-\frac{\partial\phi}{\partial y},\ \beta_z=-\frac{\partial\phi}{\partial z}}$

5. $\boxed{V_x=\frac{dx}{dt}=\frac{\omega}{-\frac{\partial\phi}{\partial x}}=\frac{\omega}{k}}$

6. 相速度的投影可超越光速，但群速度不行（群速度可以承載資訊[加上調變]，相速度不行，所以無超光速通訊） $\boxed{\vec{\beta}=\hat{\beta}|\nabla\phi|}$, $\hat{\beta}=$ unit vector $=-\frac{\nabla\phi}{|\nabla\phi|}$

7. $\boxed{\frac{ds}{dt}=V_p=\frac{\omega}{|\nabla\phi|}}$ 梯度方向為最大變化率方向，此時的相速度為最小值.

NO.

DATE / /

二、TEM Waves

1、TEM wave 的电場、磁場方向躺在等相位面上，與傳播方向正交。

2、$\boxed{\vec{E}_0 \cdot \beta = 0}$ TEM wave 須滿足此關係

3、$\boxed{\vec{H} = \dfrac{\hat{\beta} \times \vec{E}}{\eta}}$ 磁場可由行進方向外積电場除上本質阻抗求得

4、$\boxed{\vec{E} = \eta \vec{H} \times \hat{\beta}}$ 电場可由本質阻抗乘上行進方向外積磁場求得

三、Phase and Group Velocities

1、相速度：$\boxed{V_p = \dfrac{\omega}{|\nabla\phi|} = \dfrac{C}{\sqrt{\mu_r \varepsilon_r}}}$

2、$\boxed{\vec{S}_{av} = Re(\vec{S}) = \dfrac{1}{2\eta} \hat{\beta} |E_0|^2}$ 正弦波信號的波印亭向量

3、$\boxed{\vec{H} = (\hat{a}_y \cos\phi - \hat{a}_x \sin\phi) \dfrac{E_0}{\eta_0} e^{-j\vec{\beta}\cdot\vec{r}}}$

4、$\boxed{V = \dfrac{\omega}{\beta}}$

5、群速度：$\boxed{\vec{V}_g = \dfrac{\vec{S}_{av}}{\text{total time avergy energy density}}}$ $\left(\dfrac{m}{s}\right)$

(群速度是波印亭向量除以平均時間能量密度的傳播速度)

NO.

DATE / /

四、Wave Impedance

$$Z_x^- = \frac{E_z}{H_y} = -\frac{\eta_0}{\cos\phi}$$

$$Z_x^+ = -\frac{\eta_0}{\cos\phi}$$

x方向的 wave impedance 投影量 (比η大)

$$Z_y^- = -\frac{E_z}{H_z} = \frac{\eta_0}{\sin\phi}$$

$$Z_y^+ = -\frac{\eta_0}{\sin\phi}$$

y方向的 wave impedance 投影量 (比η大)

Wave impedance should be realized as E and H field ratio

五、Standing Wave Ratio

1. SWR定義： $SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$, $|\Gamma| = \frac{|E_0^-|}{|E_0^+|}$ (反射係數)

六、Wave in Lossy Media

- In lossy medium:

$(\nabla^2 - \gamma^2)\psi = 0$ $\quad \gamma^2 = -\omega^2\mu\epsilon_e \quad \nabla\cdot\vec{E} = 0$

$$\boxed{\begin{cases} \psi = A(\vec{r})e^{j\theta(\vec{r})} \\ \psi = A e^{-\gamma_x x} e^{-\gamma_y y} e^{-\gamma_z z} \end{cases}}$$

$$\gamma_x^2 + \gamma_y^2 + \gamma_z^2 = \gamma^2$$

$$\theta(\vec{r}) = -\gamma_x x - \gamma_y y - \gamma_z z = \Delta(\vec{r}) + j\phi(\vec{r})$$

$$\Rightarrow \Delta(\vec{r}) = -\alpha_x x - \alpha_y y - \alpha_z z$$

$$\phi(\vec{r}) = -\beta_x x - \beta_y y - \beta_z z$$

$$\longrightarrow \begin{cases} \gamma_x = \alpha_x + j\beta_x \\ \gamma_y = \alpha_y + j\beta_y \\ \gamma_z = \alpha_z + j\beta_z \end{cases}$$

NO.

DATE / /

1. $\boxed{\vec{H}=\dfrac{\hat{r}\,r\times\vec{E}}{j\omega\mu}}$ $\rightarrow$ $\vec{H}=\dfrac{\hat{r}\times\vec{E}}{\eta_c}$ $\quad \vec{r}=\hat{r}\,r=\hat{r}(\vec{\alpha}+j\vec{\beta})$

2、 $\boxed{\eta_c=\sqrt{\dfrac{\mu}{\varepsilon_e}}}$ 耗損材料中的本質阻抗

3. $\boxed{\vec{E}=\eta_c(\vec{H}\times\hat{r})}$ E和H的關係

4. $\boxed{\vec{S}=\frac{1}{2}(\vec{E}\times\vec{H}^*)=\hat{r}\dfrac{\vec{E}\cdot\vec{E}^*}{2\eta_c^*}}=\dfrac{\hat{r}}{2\eta_c^*}|E_0|^2e^{-2\alpha\hat{r}\cdot\vec{r}}$

$\vec{r}=(\alpha+j\beta)\hat{r}=r\cdot\hat{r}$ $\quad \alpha+j\beta=\sqrt{-\omega^2\mu\varepsilon_e}$

5. 假設 $\boxed{\mu''=0}$ 大部分材料皆如此

$\Rightarrow$ ① 介电材料(Good dielectric):

$$\left|\frac{\vec{J}_c}{\vec{J}_d}\right|\ll 1\Rightarrow\left|\frac{\sigma_e}{\omega\varepsilon_e}\right|\ll 1\Rightarrow \alpha=\frac{\sigma_e}{2}\sqrt{\frac{\mu'}{\varepsilon_e'}}$$

$$\beta=\omega\sqrt{\mu'\varepsilon_e'}$$

② 良導体(Good conductor):

$$\left|\frac{\vec{J}_c}{\vec{J}_d}\right|\gg 1\Rightarrow\left|\frac{\sigma_e}{\omega\varepsilon_e}\right|\gg 1\Rightarrow \alpha=\beta=\sqrt{\frac{\omega\mu'\sigma_e}{2}}$$

6. 集膚深度(Skin depth): $\boxed{\left|\vec{E}(y=\frac{1}{\alpha})\right|=E_0e^{-1}}$

NO.

DATE / /

七、Polarization

<1> Linear Polarization:

$$\begin{cases} \text{if } E_{x0}=0 \rightarrow \vec{E}(z,t)=\hat{a}_y E_{y0}\cos(\omega t-\beta z+\phi_y) & \text{"y polarized"} \\ \text{if } E_{y0}=0 \rightarrow \vec{E}(z,t)=\hat{a}_x E_{x0}\cos(\omega t-\beta z+\phi_x) & \text{"x polarized"} \end{cases}$$

in phase: $\vec{E}(z,t)=\hat{a}_x E_{x0}\cos(\omega t-\beta z+\phi_x)+\hat{a}_y E_{y0}\cos(\omega t-\beta z+\phi_y)$

$\boxed{\phi_x=\phi_y=\phi} \Rightarrow \vec{E}(z,t)=(\hat{a}_x E_{x0}+\hat{a}_y E_{y0})\cos(\omega t-\beta z+\phi)$

稱為 ψ 方向上的線性極化(linear polarized)

$$\boxed{\psi=\tan^{-1}\left(\frac{E_{y0}}{E_{x0}}\right)}$$

<2> Circular Polarization:

1、圓極化波的電場端点依圓形軌跡旋転，且电場大小為定值、方向持續依圓形軌跡旋転。

2、定义:「右手圓極化」(RHCP) → 往傳播方向看过去，右手圓極化的电場依順時針方向旋転，且电場向量端点的旋轉軌跡需呈圓形。

「左手圓極化」(LHCP) → 往傳播方向看过去，左手圓極化的电場依逆時針方向旋轉。

3、RHCP: $|\vec{E}|$ is a constant, $\vec{E}(z,t)=\mathrm{Re}[\vec{E}(z)e^{j\omega t}]$

$\vec{E}(z)=\hat{a}_x E_R e^{j(-\beta z+\phi_x)}+\hat{a}_y E_R e^{j(-\beta z+\phi_x-\frac{\pi}{2})}\boxed{=E_R e^{j\phi_x}(\hat{a}_x-j\hat{a}_y)e^{-j\beta z}}$

$$\rightarrow \begin{cases} |E_x|=|E_y| \\ \angle E_y=\angle E_x-90^\circ \end{cases}$$

4. LHCP: $|\vec{E}|$ is a constant,

$\boxed{\vec{E}(z)=E_L e^{j\phi_x}(\hat{a}_x+j\hat{a}_y)e^{-j\beta z}} \rightarrow \begin{cases} |E_x|=|E_y| \\ \angle E_y=\angle E_x+90^\circ \end{cases}$

NO.

DATE / /

<3> Elliptical Polarization:

1. 定义: $\delta = \phi_y - \phi_x$ (相位差)

$\gamma = \tan^{-1}\left(\frac{E_y}{E_x}\right)$ (場強比角度)

$\Rightarrow -180^\circ < \delta < 180^\circ$, $0 < \gamma < 90^\circ$

左手 右手

① if $\delta = 0, \pi, n\pi$ ⇒ 線性極化

② if $\delta = \frac{\pi}{2} + n\pi$ ⇒ 圓形極化

③ 若非 ①、② ⇒ 橢圓極化 (Elliptical Polarization)

2. $$\boxed{\left(\frac{E_x}{E_{x0}}\right)^2 + \left(\frac{E_y}{E_{y0}}\right)^2 - 2\left(\frac{E_x}{E_{x0}}\right)\left(\frac{E_y}{E_{y0}}\right)\cos\delta = \sin^2\delta}$$

(傾斜的) 橢圓關係式 Tilted ellipse

3. $$\boxed{\left(\frac{E_x}{E_{x0}}\right)^2 + \left(\frac{E_y}{E_{y0}}\right)^2 = 1}$$

正擺的橢圓

Fixed ellipse

⇒ 定義: 軸比(AR) = axial Ratio $= \pm \frac{\text{Major Axis}}{\text{Minor Axis}}$

+ ⇒ for LHP

− ⇒ for RHP , $1 \le |AR| \le \infty$

(圓) (線)

NO.

DATE / /

§CH5 Reflection and Transmission (反射與透射)

前言：正交/平行極化對於電磁波在材質介面上的分析特別重要，因為正交/平行極化在介面上的反射/透射特性不同。

(垂直)

一、Perpendicular Polarization (正交極化)

<1> 正交極化於無損耗介質

(介質與介質界面之TE波斜向入射)

入射波	反射波	透射波
$\vec{E}_i = \hat{a}_y E_{i0} e^{-j\vec{\beta}_i\cdot\vec{r}}$	$\vec{E}_r = \hat{a}_y E_{i0} \Gamma_\perp^b e^{-j\vec{\beta}_r\cdot\vec{r}}$	$\vec{E}_t = \hat{a}_y E_{i0} T_\perp^b e^{-j\vec{\beta}_t\cdot\vec{r}}$
$\vec{H}_i = \frac{\hat{\beta}_i\times\vec{E}_i}{\eta_1} = (\hat{a}_z\sin\theta_i - \hat{a}_x\cos\theta_i) \times \frac{E_{i0}}{\eta_1} e^{-j\vec{\beta}_i\cdot\vec{r}}$	$\vec{H}_r = \frac{\hat{\beta}_r\times\vec{E}_r}{\eta_1} = (\hat{a}_z\sin\theta_r + \hat{a}_x\cos\theta_r) \times \Gamma_\perp^b \frac{E_{i0}}{\eta_1} e^{-j\vec{\beta}_r\cdot\vec{r}}$	$\vec{H}_t = \frac{\hat{\beta}_t\times\vec{E}_t}{\eta_2} = (\hat{a}_z\sin\theta_t - \hat{a}_x\cos\theta_t) \times T_\perp^b \frac{E_{i0}}{\eta_2} e^{-j\vec{\beta}_t\cdot\vec{r}}$
$\vec{\beta}_i\cdot\vec{r} = x\sin\theta_i + z\cos\theta_i$	$\vec{\beta}_r\cdot\vec{r} = \hat{a}_x\sin\theta_r - \hat{a}_z\cos\theta_r$	$\vec{\beta}_t\cdot\vec{r} = x\sin\theta_t + z\cos\theta_t$

<2> Boundary Conditions (邊界條件)

1、$\hat{n}\times(\vec{E}_2-\vec{E}_1)=0 \Rightarrow \boxed{E_{ty} - E_{iy} - E_{ry} = 0}$

$$\Rightarrow e^{-j\beta_1 x\sin\theta_i} + \Gamma_\perp^b e^{-j\beta_1 x\sin\theta_r} = T_\perp^b e^{-j\beta_2 x\sin\theta_t} \quad \text{—— ①}$$

切線方向電場在邊界上的關係式

2、$\hat{n}\times(\vec{H}_2-\vec{H}_1)=0 \Rightarrow \boxed{H_{tx} - H_{ix} - H_{rx} = 0}$

$$\Rightarrow \frac{1}{\eta_1}(\cos\theta_i e^{-j\beta_1 x\sin\theta_i} - \Gamma_\perp^b\cos\theta_r e^{-j\beta_1 x\sin\theta_r}) = \frac{T_\perp^b}{\eta_2}\cos\theta_t e^{-j\beta_2 x\sin\theta_t} \quad \text{—— ②}$$

切線方向磁場在邊界上的關係式

Mark: $E_{i0}\cdot\Gamma_\perp^b = E_{r0}$ $\Gamma_\perp^b = \Gamma_{TE}$

$E_{i0}\cdot\tau_\perp^b = E_{t0}$ $\tau_\perp^b = \tau_{TE}$

NO.

DATE / /

3. 利用反射定律與折射定律

$$\boxed{\beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t}$$

Snell's law $\theta_i = \theta_r$ Snell's law $\beta_1\sin\theta_i = \beta_2\sin\theta_t$

簡化①、②式 ⇒
$$\begin{cases} 1+\Gamma_\perp^b = \tau_\perp^b & \text{③} \\ \cos\theta_t \dfrac{\tau_\perp^b}{\eta_2} = \cos\theta_i \dfrac{(1-\Gamma_\perp^b)}{\eta_1} & \text{④} \end{cases}$$

4. 由③、④式，解得：

$$\boxed{\begin{aligned} \Gamma_\perp^b &= \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \\ \tau_\perp^b &= \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \end{aligned}}$$

正交極化的反射係數與透射係數

5. from Snell's Law of Reflection:

$$\sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i \Rightarrow \boxed{\cos\theta_t = \sqrt{1-\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2}}$$

入射角與兩邊材質決定後，可算出折射角

6. Normal Incidence (垂直入射)

$$\boxed{\Gamma_\perp^b\Big|_{\theta_i=0} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \qquad \tau_\perp^b\Big|_{\theta_i=0} = \frac{2\eta_2}{\eta_2+\eta_1}}$$

$E_{i0}\cdot\Gamma_{\parallel}^{b} = E_{r0}$ $\quad \Gamma_{\parallel}^{b} = \Gamma_{TM}$

Mark: $E_{i0}\cdot T_{\parallel}^{b} = E_{t0}$ $\quad T_{\parallel}^{b} = \tau_{TM}$

NO.

DATE / /

二. Parallel Polarization (平行極化)

<1> 平行極化於無損耗介質

(介質與介質界面之 TM 波斜向入射)

入射波	反射波	透射波
$\vec{E}_i = (\hat{a}_x\cos\theta_i - \hat{a}_z\sin\theta_i)E_{i0}\,e^{-j\vec{\beta}_i\cdot\vec{r}}$	$\vec{E}_r = E_{i0}\Gamma_{\parallel}^{b}(\hat{a}_x\cos\theta_r + \hat{a}_z\sin\theta_r)\cdot e^{-j\vec{\beta}_r\cdot\vec{r}}$	$\vec{E}_t = (\hat{a}_x\cos\theta_t - \hat{a}_z\sin\theta_t)\cdot E_{i0}T_{\parallel}^{b}\cdot e^{-j\vec{\beta}_t\cdot\vec{r}}$
$\vec{H}_i = \hat{a}_y\dfrac{E_{i0}}{\eta_1}e^{-j\vec{\beta}_i\cdot\vec{r}}$	$\vec{H}_r = \dfrac{\hat{\beta}_r\times\vec{E}_r}{\eta_1} = -\hat{a}_y\dfrac{E_{i0}}{\eta_1}\Gamma_{\parallel}^{b}e^{-j\vec{\beta}_r\cdot\vec{r}}$	$\vec{H}_t = \dfrac{\hat{\beta}_t\times\vec{E}_t}{\eta_2} = \hat{a}_y\dfrac{E_{i0}\cdot T_{\parallel}^{b}}{\eta_2}e^{-j\vec{\beta}_t\cdot\vec{r}}$

<2> Boundary Conditions (邊界條件) 同正交極化之推導方式

1. $\hat{n}\times(\vec{E}_2-\vec{E}_1)=0 \Rightarrow \cos\theta_i e^{-j\beta_1 x\sin\theta_i} + \cos\theta_r\Gamma_{\parallel}^{b}e^{-j\beta_1 x\sin\theta_r} = T_{\parallel}^{b}\cos\theta_t e^{-j\beta x\sin\theta_t}$ —— ①

2. $\hat{n}\times(\vec{H}_2-\vec{H}_1)=0 \Rightarrow \dfrac{e^{-j\beta_1 x\sin\theta_t}}{\eta_1} - \dfrac{\Gamma_{\parallel}^{b}e^{-j\beta_1 x\sin\theta_r}}{\eta_1} = \dfrac{T_{\parallel}^{b}e^{-j\beta_2 x\sin\theta_t}}{\eta_2}$ —— ②

3. 利用反射定律與折射定律:

$$\boxed{\beta_1\sin\theta_i = \beta_1\sin\theta_r = \beta_2\sin\theta_t}$$

簡化①、②式 $\Rightarrow \begin{cases} \cos\theta_i(1+\Gamma_{\parallel}^{b}) = T_{\parallel}^{b}\cos\theta_t & \text{—— ③} \\ \dfrac{1}{\eta_1} - \dfrac{\Gamma_{\parallel}^{b}}{\eta_1} = \dfrac{T_{\parallel}^{b}}{\eta_2} & \text{—— ④} \end{cases}$

4. 由③、④式, 解得:

$$\boxed{\begin{aligned} \Gamma_{\parallel}^{b} &= -\frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \\ T_{\parallel}^{b} &= \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} \end{aligned}}$$

平行極化的反射係數與透射係數

NO.

DATE / /

5. $$\boxed{\cos\theta_t = \sqrt{1-\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2}}$$

6. Normal Incidence (垂直入射)

$$\boxed{\Gamma_{\parallel}^{b} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \qquad T_{\parallel}^{b} = \frac{2\eta_2}{\eta_1+\eta_2}}$$ 同正交極化結果

[整理]: 比較正交/平行極化之反射&透射係數

$$\Gamma_{\perp}^{b} = \frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t} \qquad \Gamma_{\parallel}^{b} = \frac{\eta_1\cos\theta_i-\eta_2\cos\theta_t}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}$$

$$T_{\perp}^{b} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t} \qquad T_{\parallel}^{b} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i+\eta_2\cos\theta_t}$$

三、Brewster Angle (布魯斯特角)

1. Brewster Angle 為 $\boxed{\text{全透射、零反射}}$

2. $$\boxed{\theta_B = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)}$$ 只有平行極化會有 Brewster Angle 的解

Brewster Angle

NO.

DATE / /

四、Critical Angle (臨界角)

1. Critical Angle 為 $\boxed{\text{全反射、零透射}}$

2. $\boxed{\theta_c = \sin^{-1}\left(\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}\right)^{\frac{1}{2}}}$ 由高介电材料入射低介电材料才有全反射角

Critical Angle

[討論]: ① θ_c is real if $\varepsilon_2 \le \varepsilon_1 \Rightarrow |\Gamma(\theta_i)| = 1$ if $\theta_i > \theta_c$

大於全反射角入射時，都會產生全反射.

② $\theta_i < \theta_c \Rightarrow \theta_t < \frac{\pi}{2}$ 尚未達到全反射的狀況

③ $\theta_i = \theta_c \Rightarrow \theta_t = \frac{\pi}{2}$ 已達到全反射的狀況

④ $\theta_i > \theta_c$ 時， $\cos\theta_t = \pm j\left[\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)^2 - 1\right]^{\frac{1}{2}}$

大於全反射角時，透射角無真實解

五、Surface Wave (表面波)

1. 全反射角的推導顯示會有一能量流沿介面方向傳播，該能量流稱為表面波，此一物理机制與电磁波散射極為相関，也有特殊的天线形式用此一机制設計.

2. Surface wave: $\vec{\alpha}_2 \cdot \vec{\beta}_2 = 0$

$|\Gamma_\perp^b| = 1$, $\psi_\perp = \tan^{-1}\left(\frac{\eta_1\alpha_2}{\eta_2\beta_2\cos\theta_i}\right)$

3. $\vec{S}_{av}^t = \hat{a}_x \frac{|E_{i0}^b|^2 |T_\perp^b|^2}{-2\eta_2\beta_2} \beta_1 \sin\theta_i \, e^{-2\alpha_2 z}$ 透射平均功率密度

NO.

DATE / /

4. Phase velocity (相速度)

$$V_x = V_p \quad (X方向的表面波)$$

$$V_{p2} = \frac{1}{\sqrt{\varepsilon_2 \mu_2}} \quad 介質2的一般速度$$

$$V_p = V_x = \frac{V_{p2}}{\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right)} < V_{p2} \quad \text{for } \theta_i > \theta_c$$

∴ 表面波又稱為慢波

六、Wave in Lossy Media (波於耗損介質)

<1> 介質與介質界面之正向入射.

入射波	反射波	透射波
$\vec{E}_i = \hat{a}_x E_{i0} e^{-\vec{\gamma}_i \cdot \vec{r}}$	$\vec{E}_r = \hat{a}_x E_{i0} \Gamma^b e^{-\vec{\gamma}_r \cdot \vec{r}}$	$\vec{E}_t = \hat{a}_x E_{i0} T^b e^{-\vec{\gamma}_t \cdot \vec{r}}$
$\vec{H}_i = \hat{a}_y \frac{E_{i0}}{\eta_1} e^{-\vec{\gamma}_i \cdot \vec{r}}$	$\vec{H}_r = -\hat{a}_y \Gamma^b \frac{E_{i0}}{\eta_1} e^{-\vec{\gamma}_r \cdot \vec{r}}$	$\vec{H}_t = \hat{a}_y T^b \frac{E_{i0}}{\eta_2} e^{-\vec{\gamma}_t \cdot \vec{r}}$
$\vec{\gamma}_i = \hat{a}_z(\alpha_1 + j\beta_1)$	$\vec{\gamma}_r = -\hat{a}_z(\alpha_1 + j\beta_1)$	$\vec{\gamma}_t = \hat{a}_z(\alpha_2 + j\beta_2)$

關係式: $\Gamma^b = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad T^b = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

<2> Snell's Law : $\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$

<3> $\vec{\gamma}_t = (\alpha_2 + j\beta_2)(\hat{a}_x \sin\theta_t + \hat{a}_z \cos\theta_t) = \vec{\alpha}_2 + j\vec{\beta}_2$

$$\Rightarrow \boxed{\begin{cases} \vec{\alpha}_2 = \hat{a}_z S(\alpha_2 \cos\xi - \beta_2 \sin\xi) \\ \vec{\beta}_2 = \hat{a}_x \beta_1 \sin\theta_i + \hat{a}_z S(\beta_2 \cos\xi + \alpha_2 \sin\xi) \end{cases}}$$

α_2, β_2 是材料性質

$\vec{\alpha}_2, \vec{\beta}_2$ 是透射波的性質

$\vec{\alpha}_2 \cdot \vec{\beta}_2 = |\vec{\alpha}_2||\vec{\beta}_2|\cos\psi_2 \neq 0$ 兩方向間有一夾角 ψ_2

七. Summary of Reflection and Transmission
總結

反射與透射之極化特性:

1. 決定入射波的極化特性。
2. 先寫出入射波的正交與平行極化成份,
   以決定其極化特性。
3. 找出反射波與透射波的正交與平行極化成份。
4. 決定反射波與透射波的極化特性。

八. Reflection & Transmission of Multiple Interface
(多層介質的傳播現象)

1. $Z_{in}(z=0^{+}) = \eta_2$

$$Z_{in}(z=-l) = \eta_1 \frac{1+\Gamma^{b} e^{-j2\beta_1 l}}{1-\Gamma^{b} e^{-j2\beta_1 l}} = \eta_1 \frac{\eta_2 + j\eta_1 \tan(\beta_1 l)}{\eta_1 + j\eta_2 \tan(\beta_1 l)}$$

2. $$\Gamma_{in}(z=-d^{-}) = \frac{\Gamma_{12} + \Gamma_{23} e^{-j2\beta_2 d}}{1+\Gamma_{12}\Gamma_{23} e^{-j2\beta_2 d}} \quad \text{——(A)}$$

(1) 推平面法
(簡反射係數)

$\updownarrow$ 同樣

$$\Gamma_{in}(z=-d^{-}) = \frac{\Gamma_{12} + \Gamma_{23} e^{-j2\theta}}{1+\Gamma_{12}\Gamma_{23} e^{-j2\theta}} \quad \text{——(C)}$$

$\theta = \beta_2 d$

(2) 边界條件法:
寫出边界上的
电場関係,求解

(3) 射線追蹤法

NO.

DATE / /

九、Quarter-Wavelength Transformer

Assume $f_0 = 10$ GHz, $d = \frac{\lambda_{20}}{4}$

We want $\Gamma_{in}(f_0) = 0$ (反射係數=0，等同匹配)

$$\Rightarrow \text{Let } \left| \frac{\Gamma_{12} - \Gamma_{23}}{1 - \Gamma_{12}\Gamma_{23}} \right| = 0 \quad \text{(matched condition)}$$

$$\Rightarrow |\Gamma_{12} - \Gamma_{23}| = 0 \Rightarrow |\eta_2^2 - \eta_1\eta_3| = 0 \Rightarrow \boxed{\eta_2 = \sqrt{\eta_1\eta_3}}$$

$$\because \eta_1 = \eta_0 , \quad \eta_3 = \frac{\eta_0}{2} , \quad \eta_2 = \frac{\eta_0}{\sqrt{2}} \Rightarrow \varepsilon_{r2} = 2 \Rightarrow d = 0.53 \text{ cm}$$

十、Reflection Coefficient of Multiple Layers

If each slab $d_i = \frac{\lambda_{i0}}{4}$ at $f = f_0$

$$\Rightarrow \boxed{\Gamma_{in} \approx \sum_{n=0}^{N} \Gamma_n e^{-j2n\theta}} \qquad \theta = \frac{\pi}{2} \text{ at } f = f_0$$

Multiple layer 的等效反射係數